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INVESTIGATION OF INCOMPRESSIBLE CASCADE FLOWS USING A VISCOUS/INVISCID INTERACTIVE CODE

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Investigation of Incompressible Cascade Flows Using a Viscous/Inviscid Interactive Code

by

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ABSTRACT

A two dimensional, incompressible viscous inviscid interaction computer code, designed to compute cascade flows, was investigated. Comparison of the flow characteristics predicted by the code with experimentally available data indicates that the code predicts reasonably well flow parameters on lightly loaded cascades. However, the code fails to predict correctly the actual boundary layer development and the velocity distribution for highly loaded cascades. It is concluded that further improvement of the code is needed and recommendations are presented to achieve the required improvements.

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I. INTRODUCTION

The need for better and more efficient gas turbines requires the availability of cheap and reliable design tools for blades used in compressors and turbines. Computational methods are the preferred choice for such a design tool, considering the cost and complexity of wind tunnel experiments.

Among the computational methods available today, the logical choice seems to be a computer code that can solve directly the full Navier Stokes equations. However, given the state of the art in both algorithms and computer hardware, such Navier Stokes solvers are restricted only to supercomputers, and even then the computation time is quite long.

In order to enable fast and efficient computations, the viscous inviscid interaction code was developed by Cebeci [Ref. 1]. The approach used in this code is to solve the outer flow field using potential methods, and solving the boundary layer flow using a boundary layer method subject to an interaction law, that couples the inner and the outer flows. This interaction law is needed because classical boundary layer methods fail in areas of flow reversal and separation, which are very common in real life flows.

The viscous inviscid interaction code was originally developed, and successfully used, for flows about single airfoils. It was later adapted to cascade flows.

In this thesis the applicability of the code to cascades was investigated by comparing its results to experimental data. It was found that although the code can reasonably predict experimental results in some cases, it still needs improvements before it can be applied generally as a reliable design tool.

A major restriction in improving the code is the lack of a wide data base of appropriate experimental results. Some of the key elements in the code, like transition and turbulence modelling, are based on empirical correlations, and more detailed and accurate experiments should be performed, before a better understanding of these phenomena can be achieved.

In the following, the theoretical background of the code is presented in Chapter II, a description of the code in Chapter III, the results and discussions are presented in Chapter IV and the conclusions and recommendations in Chapter V. A listing of the computer program is given in Appendix A.

II. CASCADE FLOW PROBLEM FORMULATION

This chapter outlines the theoretical background of the viscous inviscid interactive method used in the computer code to investigate cascade flows. Only the major steps in the mathematical developments will be described here. A detailed description of the theory and the numerical methods is given by Cebeci and Bradshaw [Ref. 2] and by Krainer [Ref. 3] on which this chapter is based.

A. INVISCID FLOW METHOD

Inviscid flow is the first building block of the flow and is solved using the panel method. The incompressible two dimensional outer flow must satisfy the Laplace equation:

$$\nabla^2\Phi = 0 \, ,$$

subject to the boundary conditions on the surface of the blade:

$$\frac{\partial\Phi}{\partial n} = v_w \, ,$$

where the commonly used boundary condition of zero normal velocity on the surface is replaced by a specified blowing velocity v_w to allow for the effect of the boundary layer on the outer flow.

In addition, the Kutta condition must be satisfied, in order to prevent the existence of discontinuous pressure distribution near the trailing edge of the blade.

Since the Laplace equation is linear, a solution to a complex flow field can be built by superposition of solutions of elementary flows. The elementary flows used in the panel method are the uniform parallel flow and flows about singularities (sources and vortices).

The panel method is based on replacing each blade by a distribution of sources and vortices on its surface. The surface is divided into a finite number of straight segments, called panels.

On each panel, a uniform source distribution and a uniform vorticity distribution is assumed. The source strength at each panel is set to satisfy the boundary condition at the midpoint of the panel (called the control point), and so, in general the source

strength will vary from panel to panel. The vorticity strength is assumed to be the same for all the panels and is set to satisfy the Kutta condition.

The cascade is defined as an infinite row of similar blades, each one modelled by panels of source and vortex distributions. The flow at each point is found by summing the contributions of all the singularities on all the blades , and the uniform parallel flow.

A useful concept in dealing with such flows is the concept of influence coefficients. An influence coefficient is defined as the velocity at a point induced by a unit strength singularity placed at some other field point. The influence coefficients are a function of geometry and so can be computed for a given cascade and a given choice of panel geometry.

Using the influence coefficients, the normal and tangential velocities at each control point can be written as a function of the unknown source strength of each panel and the unknown vortex strength. Using the boundary conditions, by equating the normal velocity at each control point to the prescribed blowing velocity v_w, and using the Kutta condition (which requires equal velocities on the upper and on the lower panels at the trailing edge), a system of linear equations is constructed.

By solving the system of equations, the strength of the sources and vortices is found, and the velocities (and the pressures) can be computed everywhere in the flow field.

The velocity distribution on the surface of the blade, computed by the panel method, is used as the boundary condition for the boundary layer flow calculations.

It should be noted that in the panel method as used in the present computer code, there is no modelling of the wake, and its effect on the flow field is ignored.

B. VISCOUS FLOW METHOD

Viscous flow is the second building block of the cascade flow and it is applied to the thin boundary layer near the blade surface.

1. Boundary Layer Theory

The boundary layer concept, first suggested by L. Prandtl, assumes that the flow field can be divided into an outer flow where the viscous effects are negligible compared to inertia effects, and a thin layer close to the surface where the viscous effects cannot be neglected. The complete presentation of the boundary layer theory, and the development of the boundary layer equations, is given by Schlichting [Ref. 4].

Under the assumptions of two dimensional incompressible thin boundary layer flow, the Navier Stokes equations and the continuity equation reduce to:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 ,$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + v\frac{\partial}{\partial y}\left(b\frac{\partial u}{\partial y}\right) ,$$

with the boundary conditions:

$$y = 0 \qquad u(x,0) = 0 , v(x,0) = 0 ,$$

$$y = y_e \qquad u(x,y_e) = u_e(x) ,$$

where b denotes $1 + \frac{\nu_t}{\nu}$.

Writing the velocity components in terms of a stream function Ψ :

$$u = \frac{\partial \psi}{\partial y} ,$$

$$v = -\frac{\partial \psi}{\partial x} .$$

This eliminates the continuity equation (which the stream function satisfies by definition).

Introducing the Falkner Skan transformation:

$$\eta = \sqrt{\frac{u_e}{\nu x}}\, y ,$$

$$f(x,y) = \frac{1}{\sqrt{u_e \nu x}}\psi(x,y) ,$$

the momentum equation and the boundary conditions transform to:

$$(bf'')' + \frac{m+1}{2}ff'' + m[1 - (f')^2] = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) ,$$

$$\eta = 0 \qquad f'(x,0) = 0 , f(x,y) = 0 ,$$

$$\eta = \eta_e \qquad f'(x, \eta_e) = 1 ,$$

where m is defined by:

$$m = \frac{x}{u_e} \frac{du_e}{dx} .$$

The third order differential equation can be reduced to a system of first order differential equations by introduction of two new variables U and V :

$$U = f' ,$$

$$V = U' ,$$

$$(bV)' + \frac{m+1}{2} fV + m(1 - U^2) = x\left(U \frac{\partial U}{\partial x} - V \frac{\partial f}{\partial x} \right) ,$$

with the boundary conditions:

$$\eta = 0 \qquad U(x,0) = 0 , f(x,0) = 0 ,$$

$$\eta = \eta_e \qquad U(x, \eta_e) = 1 .$$

The next step is to use a finite difference approach to solve the equations. The box method is applied using central differencing in both the x and η directions, and satisfying the equations midway between nodes.

Applying the box method results in a system of nonlinear equations in the unknown variables (which are f, U and V in each node along the η direction at the current x station).

In order to solve the nonlinear system the Newton iterative procedure is used, linearizing the equations first about the solution at the adjacent upstream station, and then about the preceding iteration. The linearization is performed by letting:

$$f_j^{i,\kappa} = f_j^{i,\kappa-1} + \delta f_j^{i,\kappa} ,$$

$$U_j^{i,\kappa} = U_j^{i,\kappa-1} + \delta U_j^{i,\kappa} ,$$

$$V_j^{i,\kappa} = V_j^{i,\kappa-1} + \delta V_j^{i,\kappa} ,$$

where:

- i denotes location in the x direction
- j denotes location in the y (η) direction
- κ indicates the iteration counter

This linearization results in a system of linear equations for the unknown increments: $\delta f_j^{i,\kappa}$, $\delta U_j^{i,\kappa}$ and $\delta V_j^{i,\kappa}$.

This system of equations is solved repeatedly until the changes in the unknowns are small enough. Since the system is block tridiagonal, Keller's block elimination method is used.

The method described so far, is a direct boundary layer method. It can be used as long as the flow does not separate. Whenever separation or flow reversal occurs, and a zero skin friction coefficient is encountered, the equations become singular and the calculations will break down.

2. Interactive Boundary Layer Method

The interactive boundary layer method is designed to overcome the difficulties encountered at regions of flow reversal and separations. In such areas the external velocity is substantially changed by the viscous effects and can no longer be considered as a known boundary condition for the boundary layer flow.

The general approach to the solution is the same as for the direct method but, since the outer flow is unknown, the velocity at the edge of the boundary layer is written as:

$$u(x, y_e) = u_{e1} + \frac{1}{\pi} \int \frac{d}{d\xi} (u_e \delta^*) \frac{d\xi}{x - \xi} ,$$

where:

1. $u_e(x, y_e)$ is the total velocity at the edge of the boundary layer.
2. $u_{e1}(x)$ is the velocity as computed by the inviscid method.
3. $\frac{1}{\pi} \int \frac{d}{d\xi} (u_e \delta^*) \frac{d\xi}{x - \xi}$ is the Hilbert integral.

The numerical solution of the boundary layer equations follows the same steps as for the direct method, but with some changes.

The transformation of the stream function and the y coordinate uses a constant velocity u_0 as a scaling factor, and a scaled velocity w is introduced:

$$\eta = \sqrt{\frac{u_0}{\nu x}}\, y \,,$$

$$f(x,\eta) = \frac{1}{\sqrt{u_0 \nu x}}\, \psi(x,y) \,,$$

$$\mathrm{w} = \frac{u_e(x,y)}{u_0} \,.$$

Using this transformation, the boundary layer equations become a system of first order differential equations:

$$f' = \mathrm{U} \,,$$

$$\mathrm{U}' = \mathrm{V} \,,$$

$$\mathrm{W}' = 0 \,,$$

$$(b\mathrm{V})' + \frac{1}{2} f\mathrm{V} + x\mathrm{W}\frac{\partial \mathrm{W}}{\partial x} = x\left(\mathrm{U}\frac{\partial \mathrm{U}}{\partial x} - \mathrm{V}\frac{\partial f}{\partial x}\right),$$

with the boundary conditions:

$$\eta = 0 \qquad \mathrm{U}(x,0) = 0 \,, f(x,0) = 0 \,,$$

$$\eta = \eta_e \qquad \mathrm{U}(x,\eta_e) = \mathrm{W}(x,\eta_e) \,,$$

$$\mathrm{w}(x,\eta_e) = \frac{u_{e1}(x)}{u_0} + \frac{1}{\pi}\int \frac{d}{d\xi}\left(\sqrt{\frac{\nu\xi}{u_0}}\,[\mathrm{W}(\xi,\eta_e)\eta_e - f(\xi,\eta_e)]\right)\frac{d\xi}{x-\xi} \,.$$

The finite difference box method is used to solve the equations, in the same way as it was used for the direct case, but with two additions:

1. In areas of flow reversal the term $u\partial u/\partial x$ is omitted to assure stable integration (the FLARE approximation).
2. The edge velocity, W_J^i (where J denotes the edge station) which involves integration, is approximated by :

$$W_J^i = g_i + c_{ii}(W_J^i \eta_J - f_J^i) \ ,$$

where g_i and c_{ii} are obtained from the numerical approximation to the Hilbert integral (which will be presented in the next section).

By using central diffrencing to approximate the differential equations, a system of nonlinear algebraic equations is obtained for the unknown variables (which are f_j^i, U_j^i, V_j^i and W_j^i). To solve the system of equations, the system is linearized by the Newton iterative procedure, and the resulting linear system is solved (for the new unknown variables which are the increments $\delta f_j^{i,\kappa}$, $\delta U_j^{i,\kappa}$, $\delta V_j^{i,\kappa}$ and $\delta W_j^{i,\kappa}$).

The solution of the system is repeated until the change in the increments is negligible compared to the preceding iteration, and the whole process is performed again at the next downstream station.

3. Interactive Model

The interactive model is used to couple the boundary layer to the external flow. It is needed in areas where strong interaction occurs, and both the boundary layer and the outer flow must be solved simultaneously. The interaction model provides the outer boundary condition to the boundary layer calculations by adding a correction term to the external velocity computed by the inviscid flow method.

The external velocity is assumed to consist of a potential flow term ($u_{eI}(x)$) and a correction term due to viscous effects ($u_{e\delta}(x)$):

$$u_e(x) = u_{eI}(x) + u_{e\delta}(x) \, .$$

The viscous effect is obtained by a surface distribution of sources on the blade (a concept first suggested by Lighthill [Ref. 5]). The normal velocities at the surface of the blade, induced by these sources, displace the streamlines from the surface in the same way that the actual boundary layer displaces them:

$$\frac{d\delta^*(x)}{dx} = \frac{v(x, \delta^*)}{u_e(x)} \ ,$$

Where $v(x, \delta^*)$ is the normal velocity at the displaced surface.

Assuming that the surface can be approximated to be a flat plate, the normal velocity will be half the local source strength σ (x). Assuming also that the inviscid velocity does not change across the boundary layer, the local source strength will be:

$$\frac{\sigma(x)}{2} = v(x,0) = v(x,\delta^*) - \int_0^{\delta^*} \frac{\partial v}{\partial y} dy = \frac{d}{dx}(u_e\delta^*) .$$

The local horizontal velocity induced by the source distribution, is the correction term to the inviscid velocity, and can be represented by the Hilbert integral:

$$\frac{1}{\pi}\int_{x_a}^{x_b} \frac{\sigma(\xi)}{x-\xi} d\xi = \frac{1}{\pi}\int_{x_a}^{x_b} \frac{d}{d\xi}(u_e\delta^*)\frac{d\xi}{x-\xi} .$$

The integration is carried out on all the sources on the surface, since the horizontal velocity is influenced by all the sources.

The Hilbert integral is then approximated by a finite series:

$$\frac{1}{\pi}\int_{x_a}^{x_b} \frac{d}{d\xi}(u_e\delta^*)\frac{d\xi}{x-\xi} = \sum_{k=1}^{K} c_{ik}(u_e\delta^*)^k .$$

Where c_{ik} is a matrix of interaction coefficients which are functions of the geometry only (i denotes the chordwise position where $u_{e\delta}$ is evaluated and k is the location of the source which effects $u_{e\delta}$).

Since the computation of $u_{e\delta}$ involves values of δ^* downstream of the current x location, which are not known yet, these terms are taken from the previous iteration using a relaxation formula.

4. Turbulence Model

The turbulence model used here is the algebraic eddy viscosity formulation of Cebeci and Smith [Ref. 6]. According to the model used in the present computer code, the eddy viscosity ν_t is defined by two different expressions, for the inner region and for the outer region:

$$\nu_t = \left\{0.4y\left[1 - \exp\left(-\frac{y}{A}\right)\right]\right\}^2 \left|\frac{\partial u}{\partial y}\right| \gamma_{tr} \quad \text{for} \quad 0 \le y \le y_c\,,$$

$$\nu_t = \alpha\int_0^\infty (u_e - u)dy\,\gamma_{tr}\,\gamma \qquad \text{for} \quad y_c \le y \le \delta\,.$$

Where:

$$A = \frac{26\nu}{\left(\nu\dfrac{\partial u}{\partial y}\right)^{1/2}}\,,$$

$$\gamma = \frac{1}{1 + 5.5(y/\delta)^6}\,,$$

$$\alpha = \frac{0.0168}{1 - \beta\left[\dfrac{\partial u/\partial x}{\partial u/\partial y}\right]^{2.5}}\,,$$

$$\beta = \frac{6}{1 + 2R_T(2 - R_T)} \qquad \text{for} \quad R_T < 1\,,$$

$$\beta = \frac{1 + R_T}{R_T} \qquad \text{for} \quad R_T \ge 1\,,$$

$$R_T = \frac{\tau_w}{(-\overline{u'v'})_{max}}\,.$$

The distance from the wall to the point between the two regions, y_c, is chosen such that the viscosity will be continuous.

The intermittency factor, γ_{tr} is defined by:

$$\gamma_{tr} = 1 - \exp\left[-\frac{u_e^3}{G_\gamma \nu^2} R_{xtr}^{-1.34} (x - x_{tr}) \int_{x_{tr}}^{x} \frac{d\xi}{u_e} \right].$$

Where:

- R_{xtr} is the Reynolds number based on external velocity and transition location.
- G_γ is an empirical constant, originally assigned the value 1200.

Cebeci and Bradshaw [Ref. 2, p.246] described a different expression for the variable A in the inner region viscosity formula:

$$A = \frac{26\nu}{(1 - 11.8p^+)^{1/2}\left(\nu \frac{\partial u}{\partial y}\right)_{max}^{1/2}}.$$

Where:

$$p^+ = \frac{\nu u_e}{\left(\nu \frac{\partial u}{\partial y}\right)^{3/2}} \frac{du_e}{dx}.$$

This version of the turbulence model was not implemented in the original computer code. During the work on this thesis, the effect of the modified turbulence model was investigated.

A different intermittency distribution was implemented successfully by Rodi and Schonung [Ref. 7] for transition over separation bubbles. They used for G_γ the expression:

$$G_\gamma = \frac{100}{\exp(0.99Tu)}.$$

Where Tu is the turbulence level in the free flow. This intermittency model was also investigated during the work on this thesis.

5. Transition

The prediction of transition from laminar to turbulent flow is very difficult and has to rely on empirical correlations. The relation used here to predict the onset of transition is a combination of Michel's method and the e^9 method, and is given by Cebeci and Bradshaw [Ref. 2 , p. 153]:

$$R_{\theta_{tr}} = 1.174\left(1 + \frac{22400}{R_{e_{xtr}}}\right)R_{e_{xtr}}^{0.46}.$$

Where:

1. $R_{\theta_{tr}}$ is the Reynolds number based on the momentum thickness at the onset of transition.
2. $R_{e_{xtr}}$ is the Reynolds number based on x at the onset of transition.

In the computer code, if a laminar separation is detected before transition occurs, the onset of transition is assumed at the point of laminar separation.

III. DESCRIPTION OF THE COMPUTER CODE

The computer code used here to investigate cascade flows was written by Cebeci, and is based on the numerical formulation that was outlined in the previous chapters. In this chapter the general structure and the major subroutines of the code will be described.

A. GENERAL STRUCTURE OF THE MAIN PROGRAM

The main program reads in the cascade data (blade coordinates, spacing and stagger angle), the flow data (inlet angle and Reynolds number), and transition parameters. The transition onset on each surface of the blade can be computed by the program, or can be input by the user. The intermittency parameter G should be specified by the user.

The program then calls subroutine POTNL to compute the outer inviscid flow field for the first cycle. The output of subroutine POTNL is the external velocity distribution on the surface of the blades. This velocity distribution is then transferred to subroutine CASBLP, which calculate the boundary layer flow.

Subroutine CASBLP returns the displacement thickness distribution and the blowing velocity distribution on the blades to the main program. This data is then transferred back to subroutine POTNL to the next cycle of calculations.

The program repeats the cycles of calculations by calling the two subroutines, until the specified number of cycles is reached, or until a convergence criterion is satisfied.

B. DESCRIPTION OF THE SUBROUTINES

1. Subroutine POTNL

This subroutine solves the inviscid outer flow by using the panel method. The subroutine calculates the influence coefficients and calculates the velocities subject to the boundary conditions.

The velocities are evaluated on the displaced surface (the surface created by adding the displacement thickness to the original surface of the blade). The input to this subroutine includes the cascade geometry, the blowing velocity and the displacement thickness (for the first cycle both the displacement thickness and the blowing velocity are taken to be zero).

2. Subroutine CASBLP

This subroutine, called by the MAIN program, receives the blade geometry and the velocity distribution as input.

It transforms the x,y blade coordinates to the chordwise tangential coordinates and smooths the velocity data (during the work on this thesis it was found that smoothing the velocity data prevents the detection of the separation bubble near the leading edge, and therefore it was eliminated). The subroutine then calls subroutine COMPBL for further calculations.

3. Subroutine COMPBL

This subroutine finds the stagnation point and controls the generation of the boundary layer calculation grid for each surface (the grid starts at the stagnation point and includes 91 points in the chordwise direction for the upper surface and 71 points on the lower surface).

The subroutine then calls subroutine BL2D which calculates the boundary layer parameters for each surface (BL2D is called twice, first for the upper surface and then for the lower surface).

4. Subroutine BL2D

This subroutine computes the displacement thickness and the blowing velocity and returns them back to the calling subroutine (COMBL) in arrays compatible with the potential flow calculations (one array that contains all the points of the blade, first the lower surface starting at the trailing edge and proceeding forward, and then the upper surface, starting at the leading edge and proceeding backwards).

BL2D calls the following subroutines:

1. Subroutine INPUT which calculates the following:
   a. NS, the switching point between direct and interactive boundary layer calculations (this point is set at the first pressure peak when the blade is scanned from leading edge towards the trailing edge)
   b. NTR, transition location (only if the transition location is an input. Otherwise it is calculated by subroutine TRNS).
   c. GMTR, the distribution of the intermittency factor γ_{tr}.

   In addition this subroutine generates the boundary layer grid in the η direction and the initial velocity profile, by calling subroutine INTL.
2. CALCIJ, calculates the c_{ij} coefficients used in the Hilbert integral approximation.
3. EDDY, calculates the eddy viscosity (called only after transition has been detected).

4. COEFTR, calculates the coefficients of the boundary layer finite difference equations in transformed form (for the direct method calculations).
5. SOLVE3, solves the linearized boundary layer equations for the F,U and V variables by computing the increments δF, δU and δV

The subroutine then checks the convergence of the Newton iterations and repeats the calculations if needed. If the subroutine detects flow separation or if it reaches the switching point NS, subroutine MAIN2 is called for the interactive method calculations. Otherwise, the subroutine proceeds to the next chordwise point of the grid (NX) and repeats the calculations.

5. Subroutine MAIN2

This subroutine calculates the boundary layer parameters by the interactive method. The subroutine performs the following steps:

1. It first calls subroutines JOINT and COMGI to compute the interaction coefficients.
2. In regions of laminar flow it calls the following subroutines:
   a. COEF, which calculates the coefficients of the boundary layer finite differences equations.
   b. SOLV4, solves for the variables F, U, V and W by computing the increments δF , δU , δV and δW .
   c. TRANS, to check if the condition for transition is satisfied (it also checks for laminar separation and initiates transition at the point of laminar separation if it is detected).

   The subroutine then checks for convergence of Newton iterations and repeats the calculations as needed.
3. In regions of turbulent flows the subroutine calls the following subroutines:
   a. EDDY, to compute the eddy viscosity parameter B $(B = 1 + \nu_t/\nu)$.
   b. COEF and SOLV4, the same as for laminar flow.

6. Subroutine OUTPUT

This subroutine computes the boundary layer parameters. It is called with a parameter "INDEX" which determines the type of calculations:

1. For INDEX = 1 the computations relates to transformed coordinates (direct boundary layer method) using the relations:

$$c_f = \frac{2\,V(1,2)\,B(1,2)}{\sqrt{R_{ex}}} ,$$

$$V_w = u_e\sqrt{\frac{u_e}{x}}\,V(1,2)\,,$$

$$D = u_e\delta^*\sqrt{R_e}\,,$$

$$\delta^* = \frac{x}{\sqrt{R_{ex}}}\left(\eta(NP) - F(NP)\right),$$

$$\theta = \frac{x}{\sqrt{R_{ex}}}\sum_{j=2}^{NP} a_j\,U_j\,(1 - U_j)\,.$$

Where V(1,2) and B(1,2) are the velocity gradient and the viscosity parameter at the surface, respectively, and η(NP) and F(NP) are η and F evaluated at the edge of the boundary layer.

2. For INDEX=2 the subroutine calculates the boundary layer parameters for semi--transformed coordinates (interactive boundary layer method) using the relations:

$$C_f = \frac{2\,V(1,2)\,B(1,2)}{\sqrt{x}\,\sqrt{R_e}\,[W(NP)]^2}\,,$$

$$V_w = \frac{V(1,2)}{\sqrt{x}}\,,$$

$$u_e = U(NP)\,,$$

$$D = (U\eta - F)\sqrt{x}\,,$$

$$\delta^* = \left(\eta - \frac{f}{U}\right)\sqrt{x}\,.$$

For NX > NTR (after transition has been detected), subroutine SMPSON is called (subroutine SMPSON calculates the coefficient of the outer region eddy viscosity). The subroutine then prints out the velocity profiles at the required stations.

7. Subroutine TRANS

This subroutine calculates the transition location based on the Michel criterion or based on laminar separation (whichever occurs first). If transition has been detected the intermittency distribution is calculated for all the remaining points of the surface.

8. Subroutine FILLUP

This subroutine increases the number of points in the boundary layer grid (in the η direction) as needed. It also fills up the arrays of F, U, B, W and V between the edge of the boundary layer to the end of the arrays (with V=0, W,B and U, with the last values that they had in the edge of the boundary layer and F as the integral of U).

9. Subroutine EDDY

This subroutine calculates the eddy viscosity using the Cebeci--Smith two layer eddy viscosity formula. It receives the vectors U,V and η at a point and computes the viscosity vector B.

10. Subroutine INTL

This subroutine generates the boundary layer grid in the η direction. It sets the number of grid points and generates the initial velocity profile.

IV. RESULTS AND DISCUSSION

The viscous inviscid interaction code was run with several cascades on which experimental data is available. In order to enable a thorough comparison between experimental results and the computed results, a very detailed experimental data base is needed. The data should include measurements of the boundary layer development along the blade, velocity profiles along the boundary layer, transition location and distribution, flow separation, and external velocity distribution.

Unfortunately, very few cascade experiments has been performed, which obtained the required data with sufficient accuracy, due mostly to the lack of appropriate measurement equipment. Only recently, with the introduction of non--interfering methods like the Laser Doppler Velocimeter (LDV), the required data can be measured accurately.

Recently an experiment involving the investigation of a linear compressor cascade of Controlled Diffusion Blading (which will be referred here as the CD cascade) has been carried out by Elazar [Ref. 8]. Most of the work in the present thesis, involves comparison of the computer code results with Elazar's experimental results.

Other cascades that were investigated are:

1. A shockless, supercritical airfoil cascade, designed in 1974 by Korn in cooperation with Pratt & Whitney Aircraft (referred here as the P & W cascade). The experimental results of the cascade were obtained from a report by Hobbs, Wagner, Dannenhoffer and Dring [Ref. 9].

2. Stator blade of a single stage axial compressor (referred here as the C4 cascade). The blade profile is the British C4 section (10% thickness) on a circular arc camber line. The experiment has been performed by Walker [Ref. 10]. The detailed boundary layer measurements are not presented in the report and were obtained directly from the author.

The code failed to run with two other cascades:

1. A highly loaded, double circular arc blade with a sharp leading edge and a sharp trailing edge, used in a compressor cascade that was investigated by Deutsch and Zierk [Ref. 11].

2. V2 double circular arc blade, highly loaded cascade. This cascade was investigated by Hoheisel and Seyb [Ref. 12].

In both cases the code calculated the potential flow successfully but failed in trying to compute the first cycle of the boundary layer calculations.

A. CD CASCADE

The experimental data for the CD cascade was obtained at $M = 0.25$, $R_e = 700000$ and at three inlet angles: 40° (the design condition), 43.4° and 46°. The spacing was 0.6 of the chord and the stagger angle 14.27°. A general layout of the cascade is shown in Figure 1 on page 20.

The following observations were concluded from the experiment:

1. A separation bubble exists near the leading edge on the upper surface at all the inlet angles. The bubble became larger at increased inlet angles.
2. Transition from laminar to turbulent flow occurred above the separation bubble (on the upper surface).
3. Transition on the lower surface occurred at midchord.
4. The boundary layer thickness on the upper surface increased with inlet angle, and reached a thickness of 15% chord at the highest inlet angle. The boundary layer thickness on the lower surface did not change significantly with inlet angle.
5. The turbulent boundary layer on both surfaces remained fully attached at all the inlet angles.

1. Transition location and intermittency distribution

The effects of the transition location and the intermittency factor were investigated. The code was first run with the transition location calculated by the code, and with several values of the intermittency factor G_γ. It was found that the code did not run with $G_\gamma = 1200$ (which is the value used usually for high Reynolds numbers). The highest value of G_γ with which the code run successfully was 900.

The code failed to predict the separation bubble on the upper surface, and predicted laminar separation at 78% chord on the lower surface (which did not occur in the experiment). Transition on the upper surface occurred at 41% chord (detected by Michel's criterion) and at 78% chord on the lower surface (at laminar separation).

The shape factor computed by the code was compared to the experimental results. As can be seen in Figure 2 on page 21 the shape factor as predicted by the code deviates substantially from the actual results, due mainly to the different transition location.

On the lower surface, as can be seen in Figure 3 on page 22 the shape factor deviates even more from the experimental results. In this figure the effect of changing the intermittency factor G_γ can be seen. For both the extreme values of G_γ, 10 and 900, the computed shape factor curve is far from agreement with the actual results.

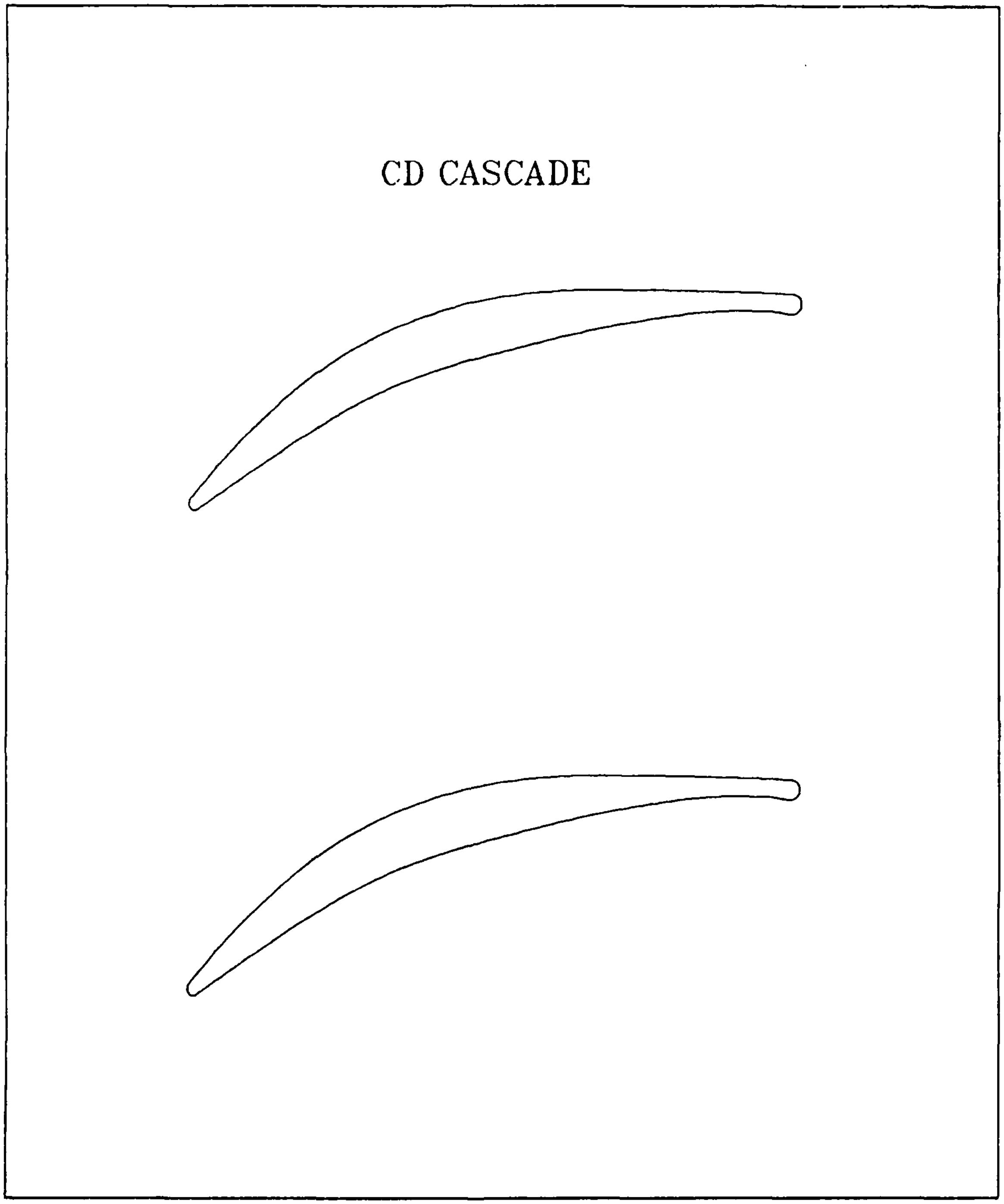

Figure 1. Controlled Diffusion cascade

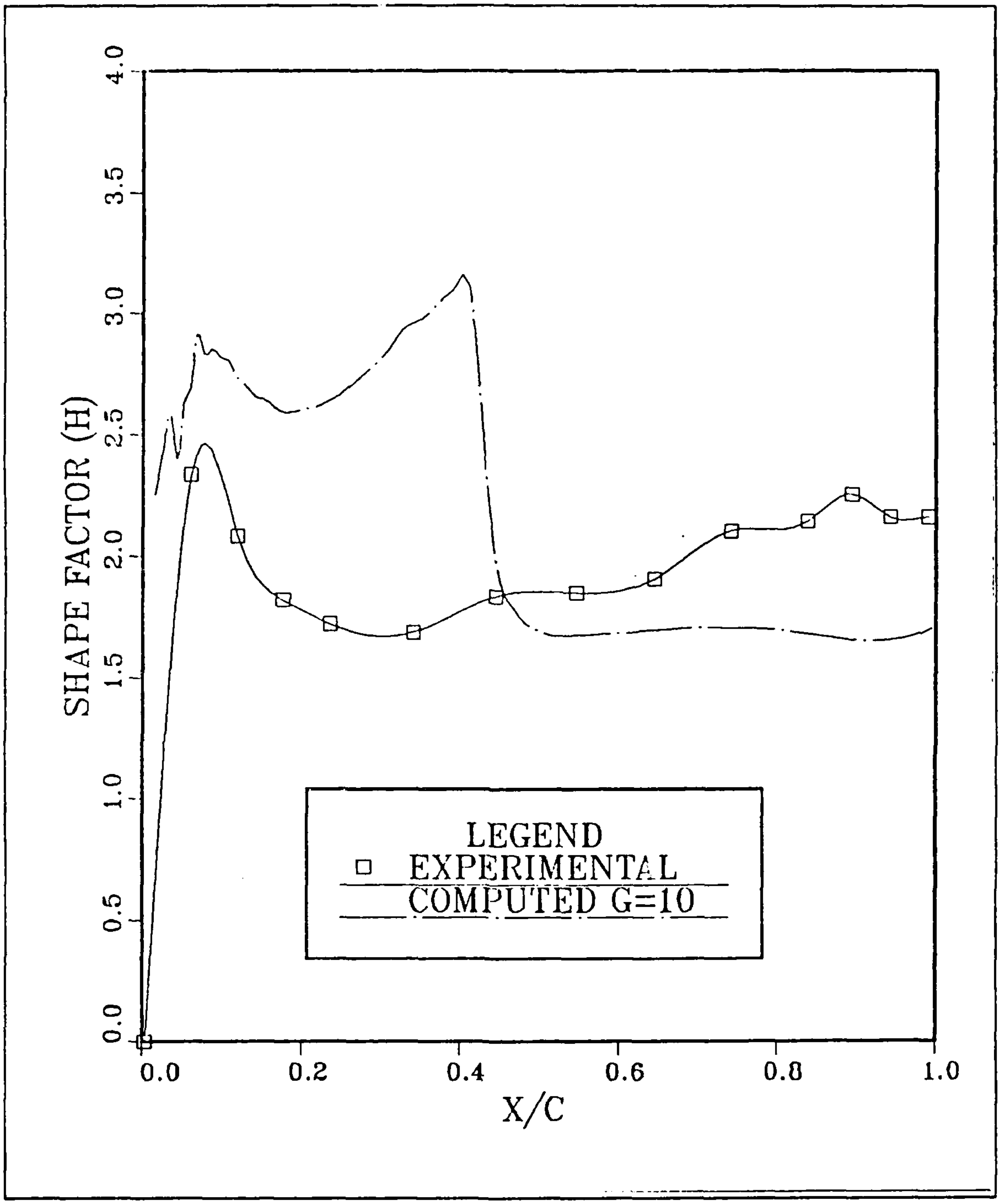

Figure 2. Shape factor comparison on the upper surface: Transition computed by the code (β = 40°)

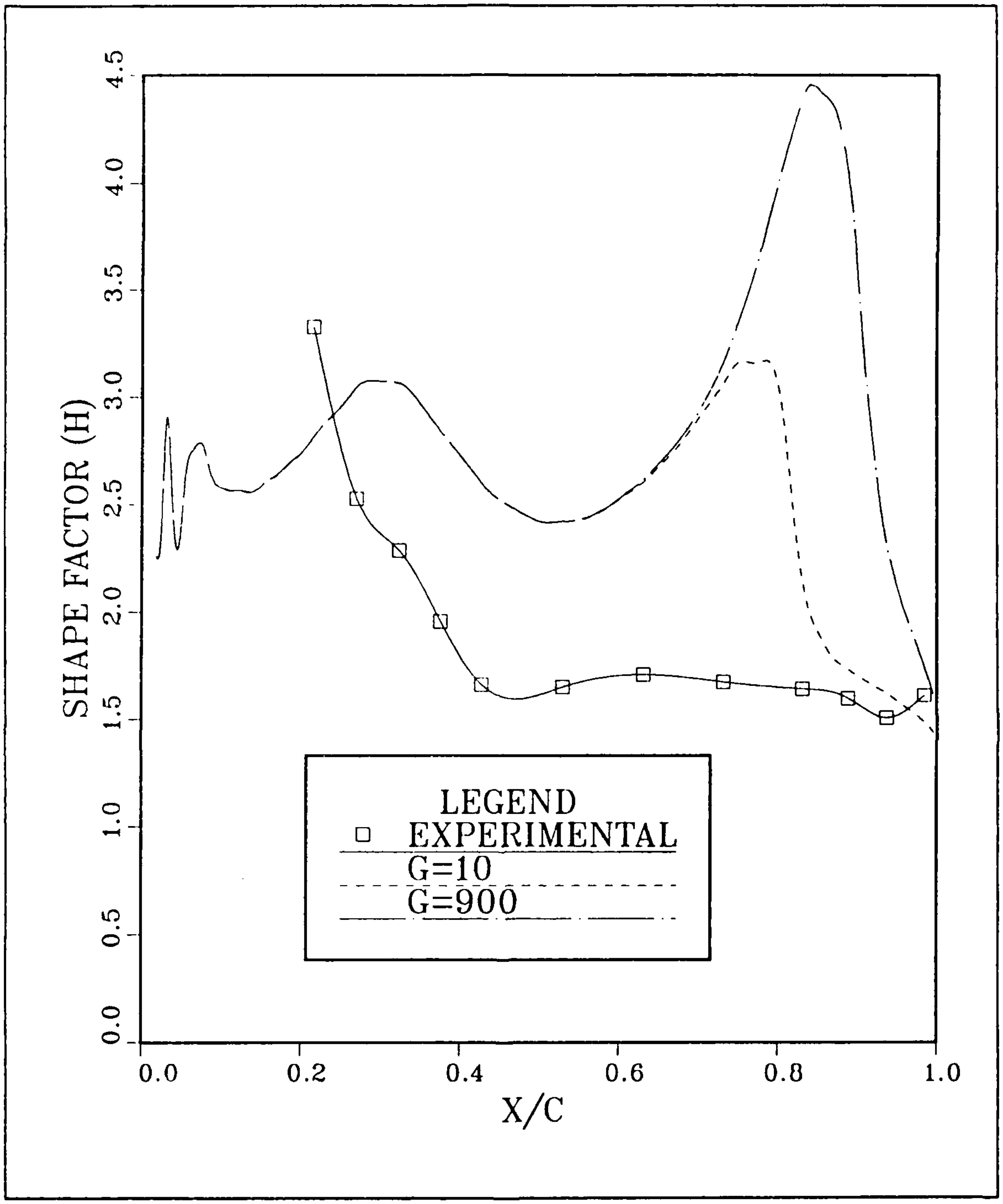

Figure 3. Shape factor comparison on the lower surface: Transition computed by the code ($\beta = 40°$).

Since the code did not predict the existence of the separation bubble on the upper surface, it was decided to try to eliminate the smoothing of the external velocity as computed by the potential flow subroutine (originally, the velocities were smoothed in subroutine CASBLP prior to boundary layer calculations). It was found that without smoothing the velocities a small separation bubble is predicted by the code at 4% of chord. The onset of transition is set by the code at the beginning of the separation bubble.

The code was run with unsmoothed velocities with two values of G_γ, 10 and 900. The shape factor behavior can be seen in Figure 4 on page 24. Changing the value of G_γ did not change the shape of the curve much, and generally the shapes of the computed and the experimental curves look alike.

The elimination of the velocity smoothing in the code, also affects the thickness of the boundary layer. In Figure 5 on page 25 the displacement thickness is plotted for both cases (with and without the velocity smoothed). Without smoothing, the displacement thickness is much thicker, especially on the rear half of the blade, which is closer to the actual results.

The effect of changing the intermittency distribution to the one used by Rodi and Schonung [Ref. 7] was investigated. It was found, as can be seen in Figure 6 on page 26 that the effect of the new model is equivalent to using G_γ in the present model.

On the lower surface it was necessary to run the code with transition as input, to get reasonable results, as can be seen in Figure 7 on page 27 for transition input at 21% of chord.

At the off design conditions (inlet angles of 43.4° and 46°) a similar behavior of the transition has been observed, as can be seen for example in Figure 8 on page 28 for the upper surface and in Figure 9 on page 29 for the lower surface, both at $\beta = 46°$.

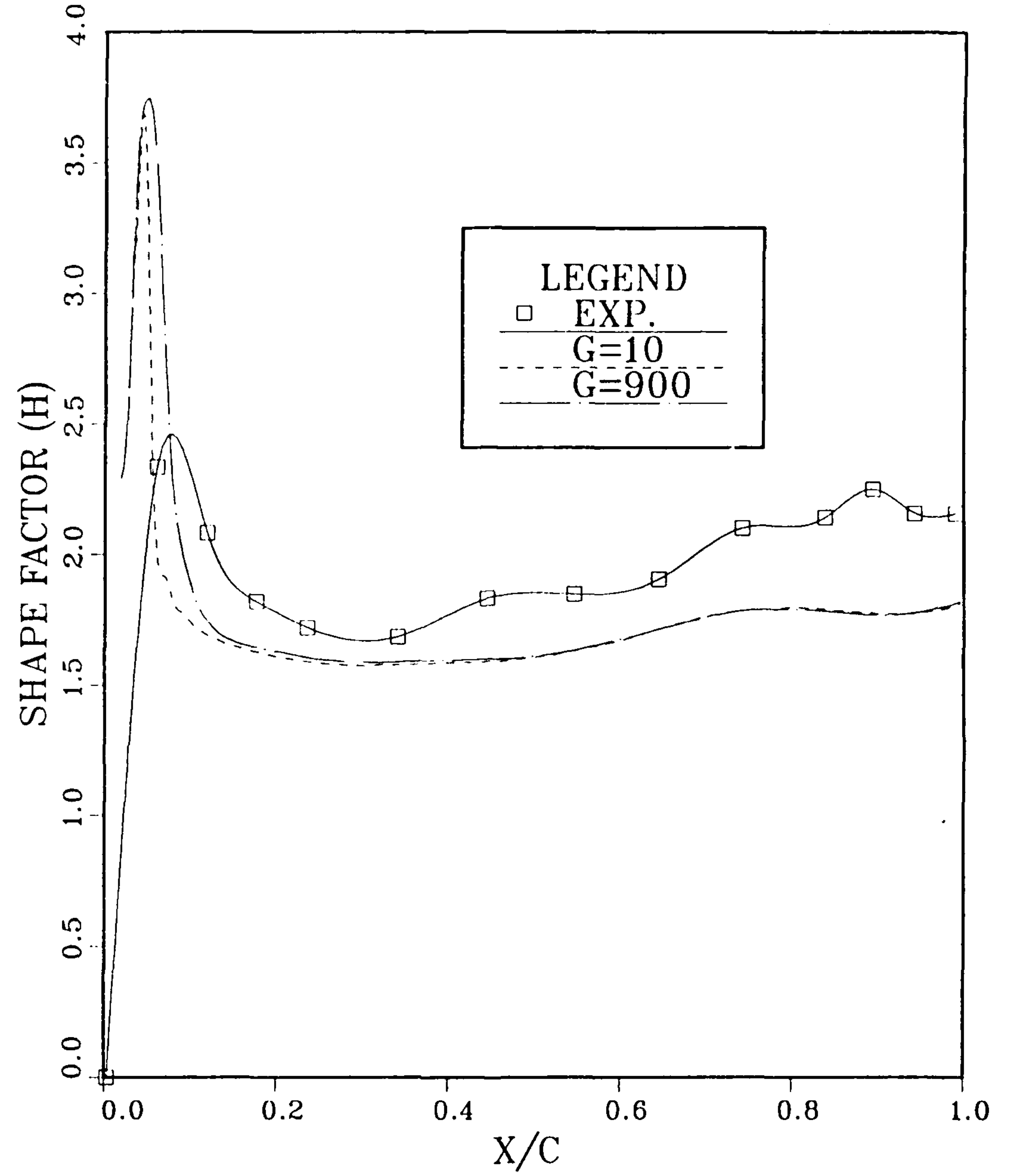

Figure 4. Shape factor on the upper surface without velocity smoothing.

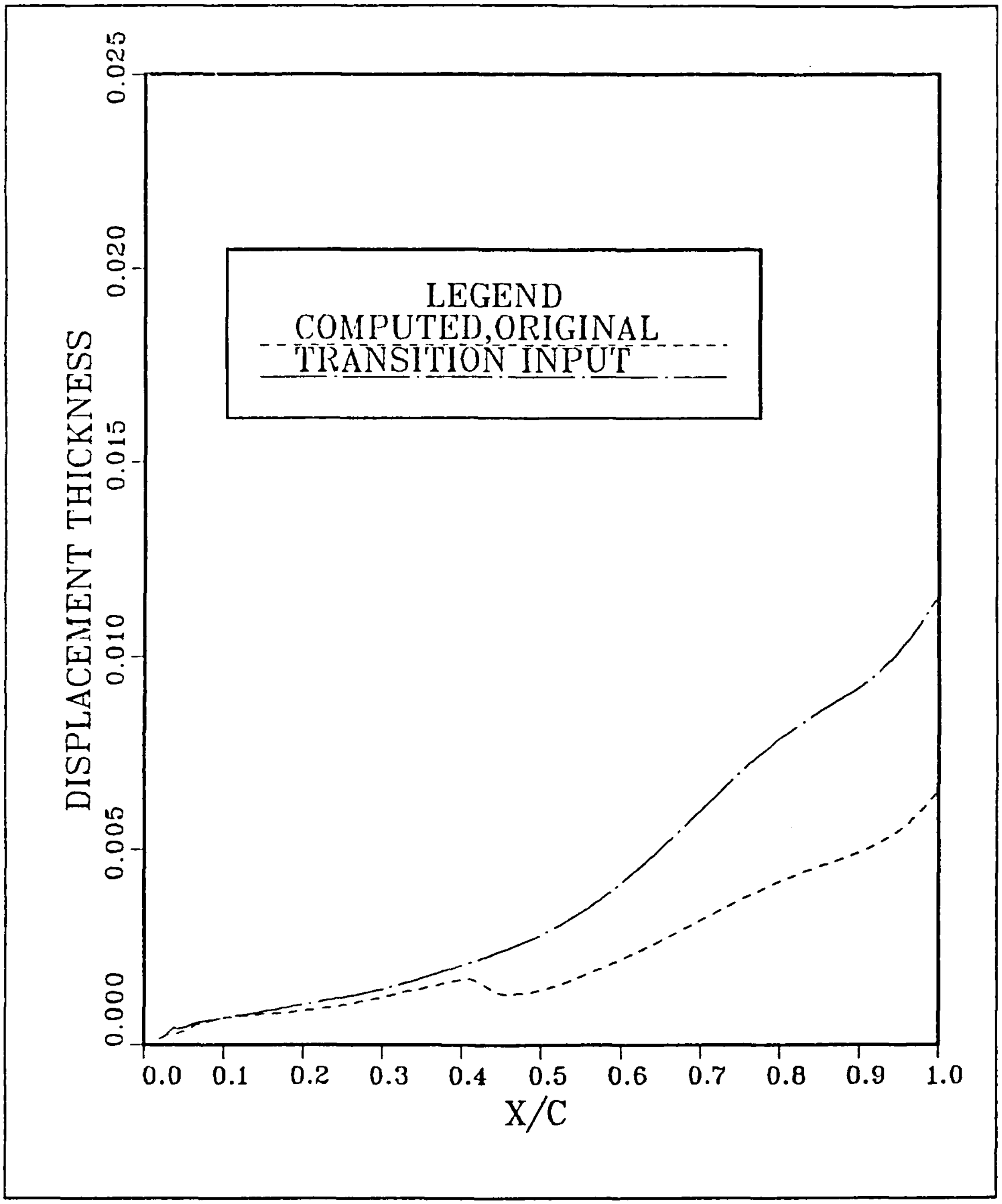

Figure 5. Displacement thickness: The effect of velocity smoothing.

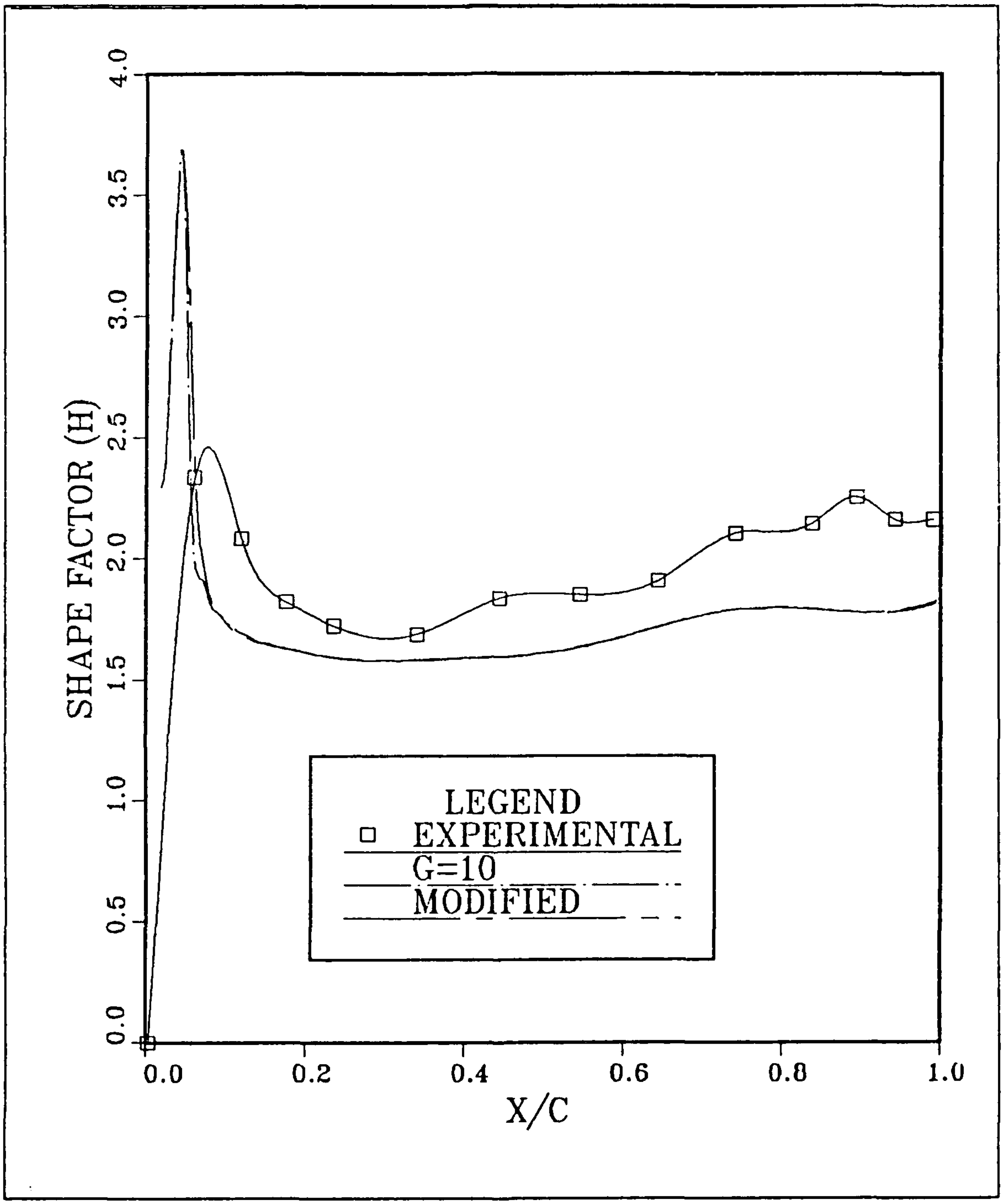

Figure 6. The effect of the intermittency model: Upper surface, $\beta = 40°$

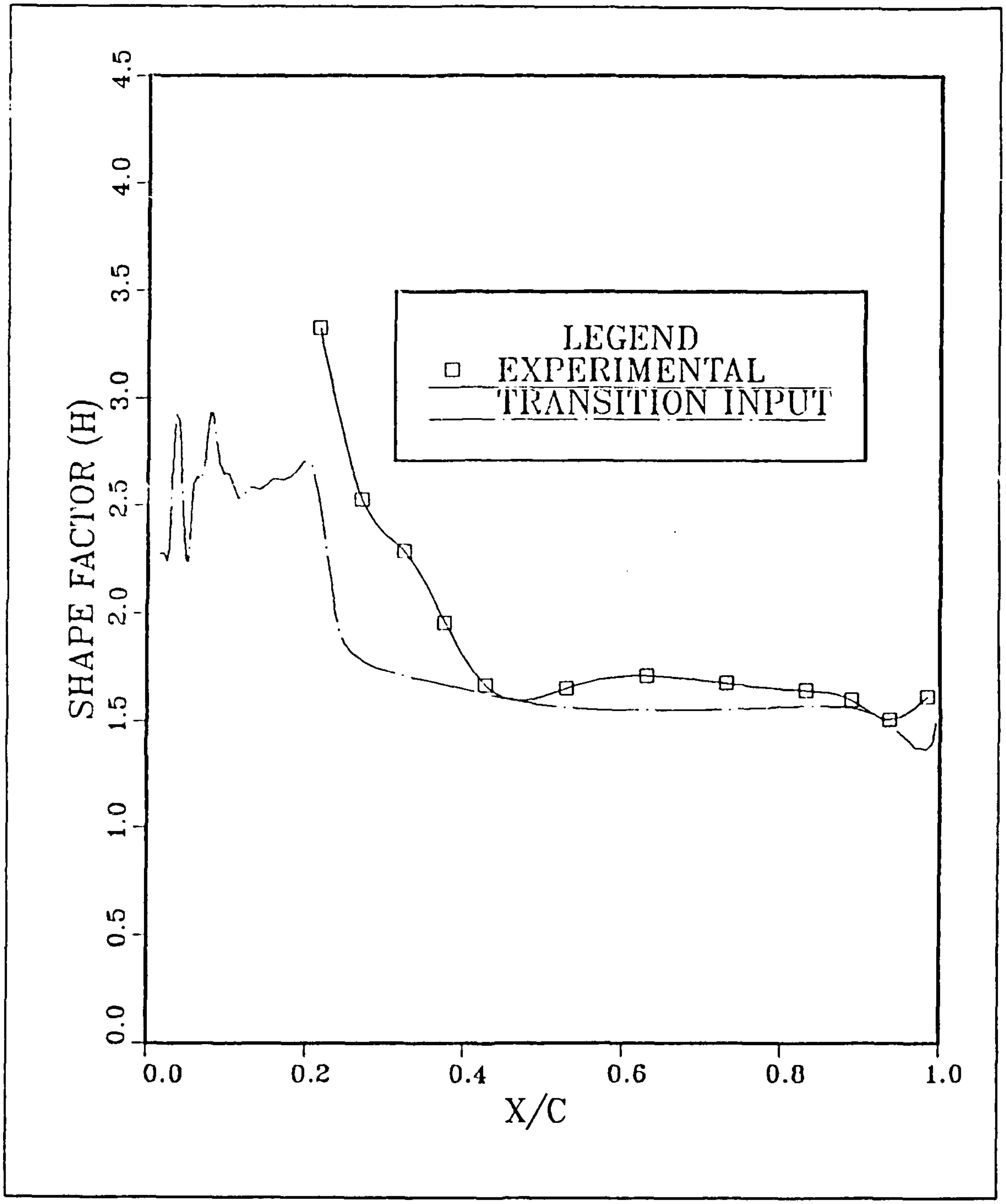

Figure 7. Shape factor on the lower surface with transition input at 21%.

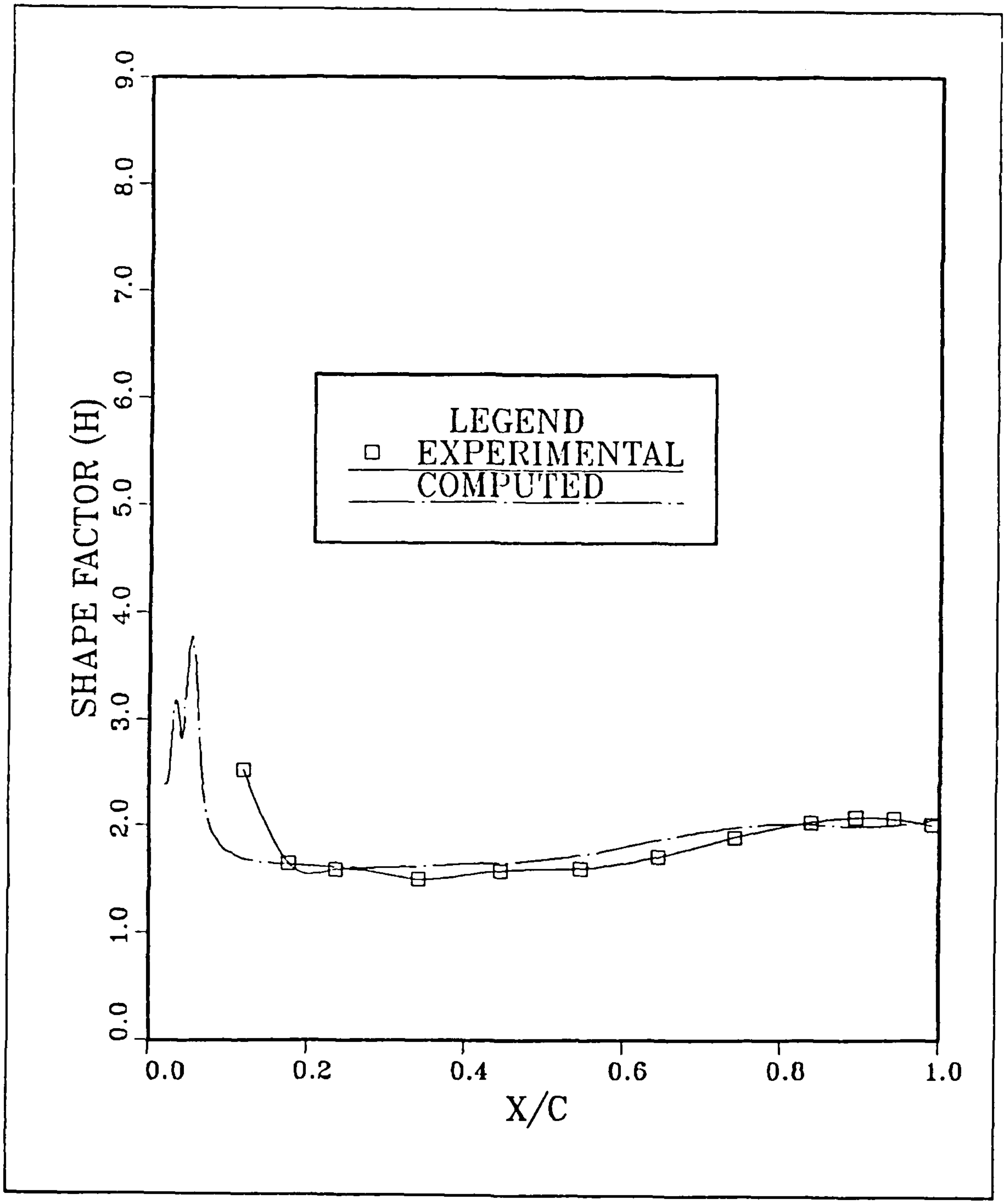

Figure 8. Shape factor at $\beta = 46°$ on the upper surface

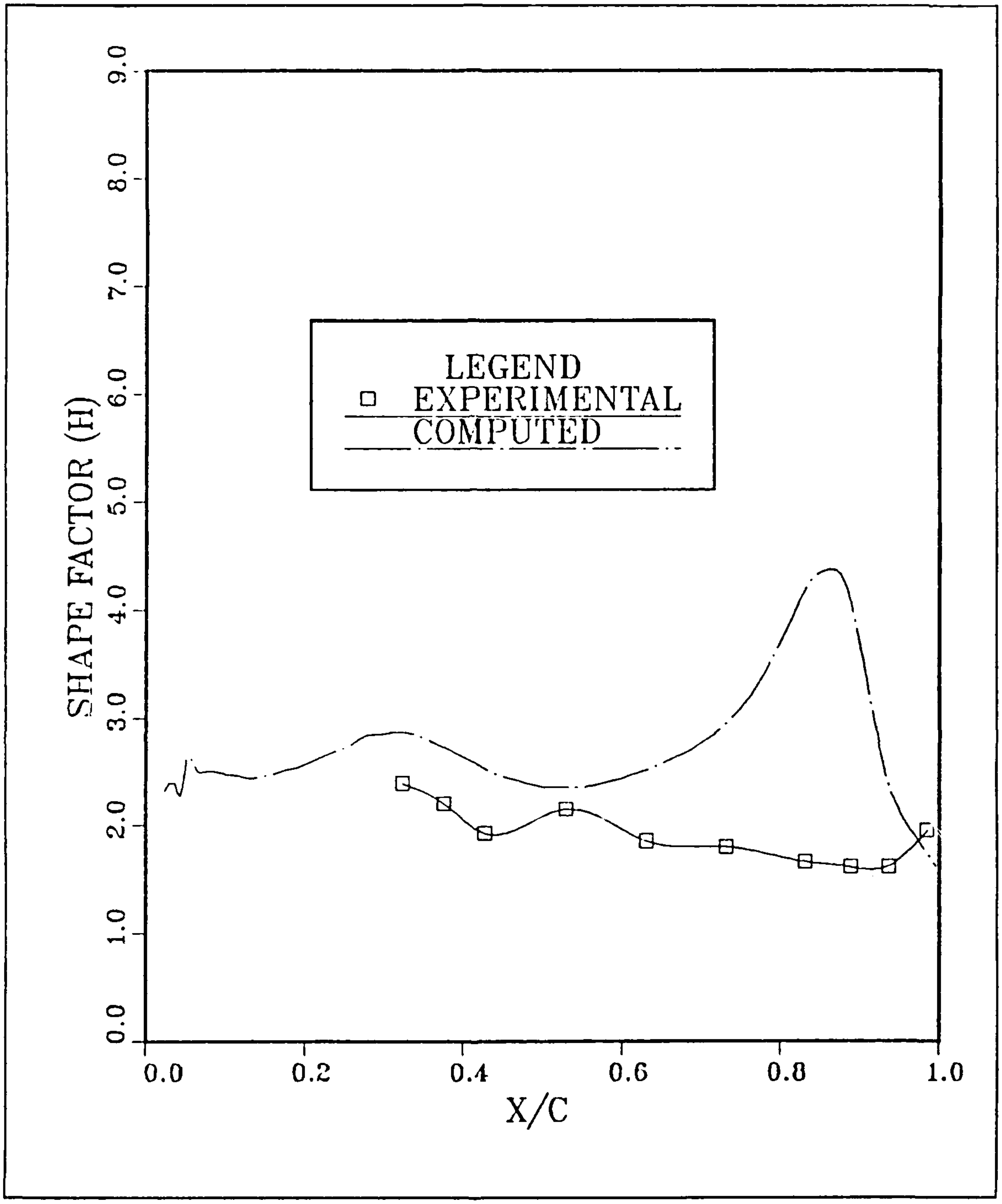

Figure 9. Shape factor at β = 46° on the lower surface.

2. External Velocity Distribution

The external velocity distribution, computed by the code using the interaction law, was compared to the experimentally measured velocities. It was found that in general, the velocities measured experimentally, were higher than those computed by the code for all the inlet angles.

There are two possible sources to the discrepancy in the velocities:

1. The computer code calculates pure 2--D flows. In the experiment the flow was observed to accelerate due to the effect of the boundary layer on the side walls (a 3--D effect). This effect was calculated in the experiment and is referred to as the AVDR correction [Ref. 8, p.43].
2. The flow accelerates due to the thickening of the boundary layer. Since the boundary layer as computed by the code is substantially thinner than the actual boundary layer (as will be discussed in the next section) the external velocities predicted by the code are smaller.

To compensate for the first error source, all the computed velocities were compared to the experimental velocities corrected by the AVDR correction. The comparison between the velocities can be seen in Figure 10 on page 31 for $\beta = 40°$, in Figure 11 on page 32 for $\beta = 43.4°$ and in Figure 12 on page 33 for $\beta = 46°$.

It can be seen from the figures that the difference between the computed and the experimental velocities is larger on the lower surface. The reason might be the method with which the correction to the inviscid velocity is computed. The assumption on which the interaction law is based, is that only sources (representing the viscous effects) on the surface being considered, affect the local velocity. In reality, the boundary layer on both surfaces affects the local velocity (because the boundary layer developed on the upper surface of a blade, causes a velocity disturbance that is felt on the lower surface of the adjacent blade).

Since the boundary layer on the lower surface is much thinner, its effect on the velocity on the upper surface is much smaller than the effect of the upper surface boundary layer on the lower surface velocity.

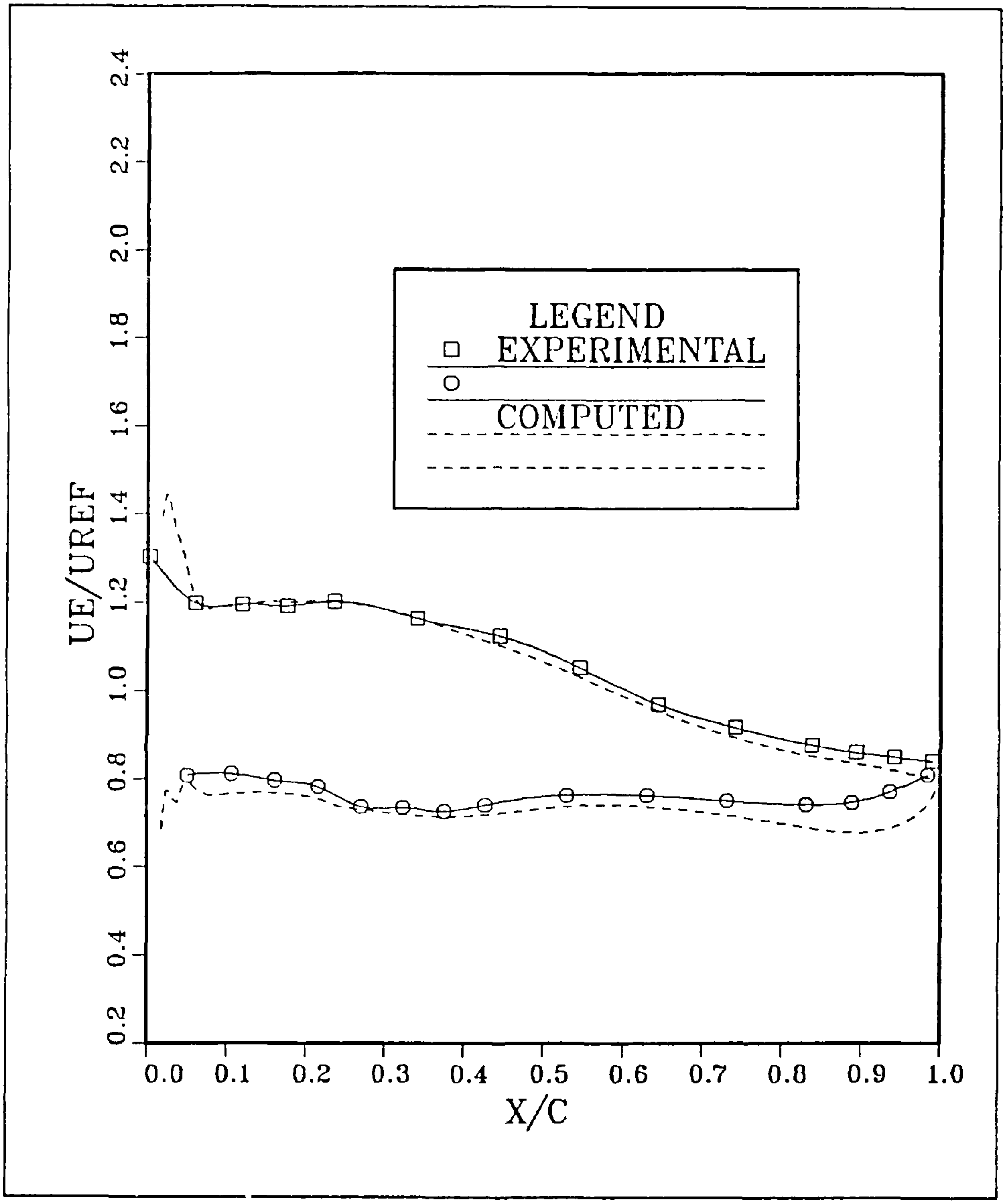

Figure 10. External velocity at $\beta = 40°$

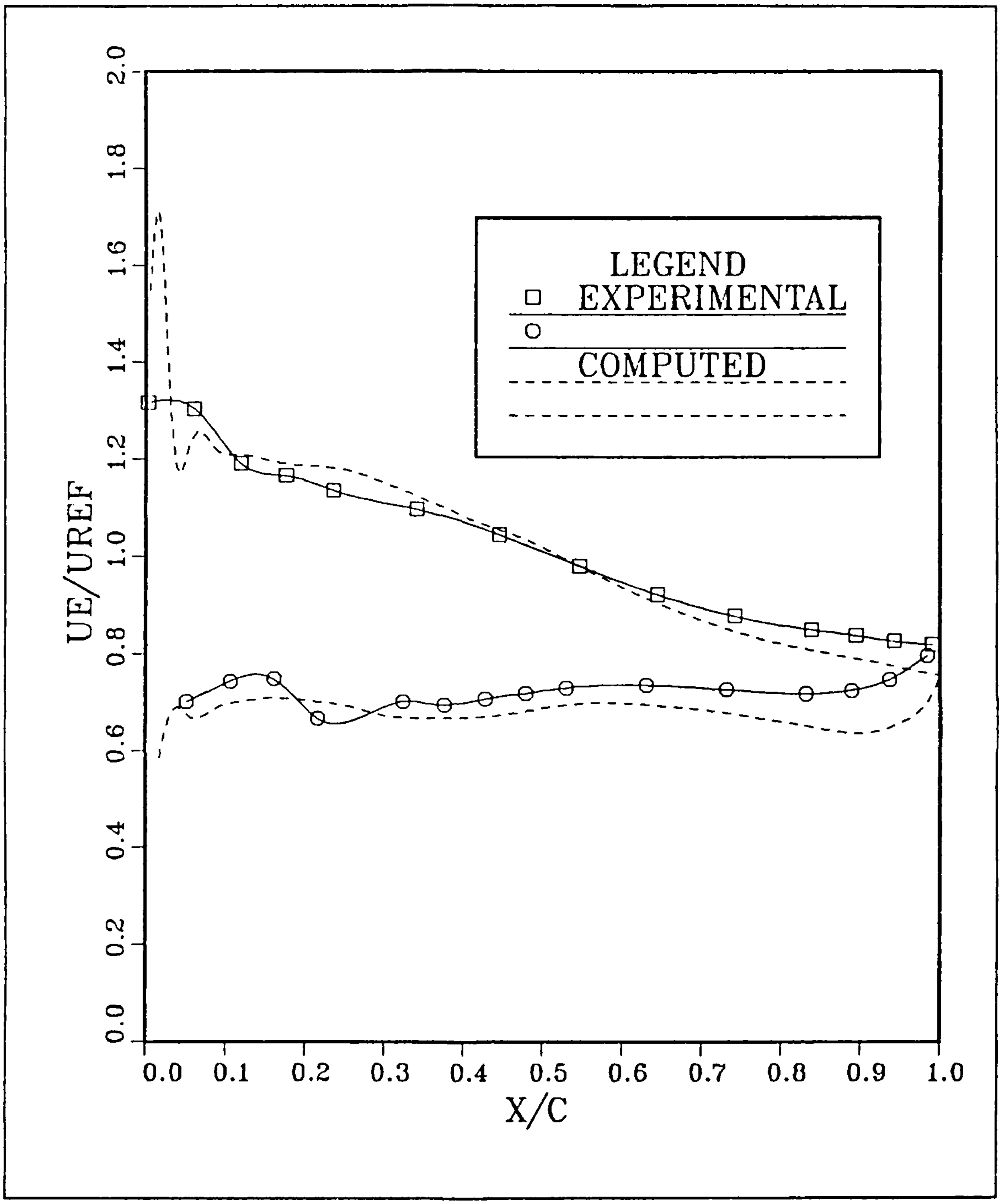

Figure 11. External velocity at β = 43.4°

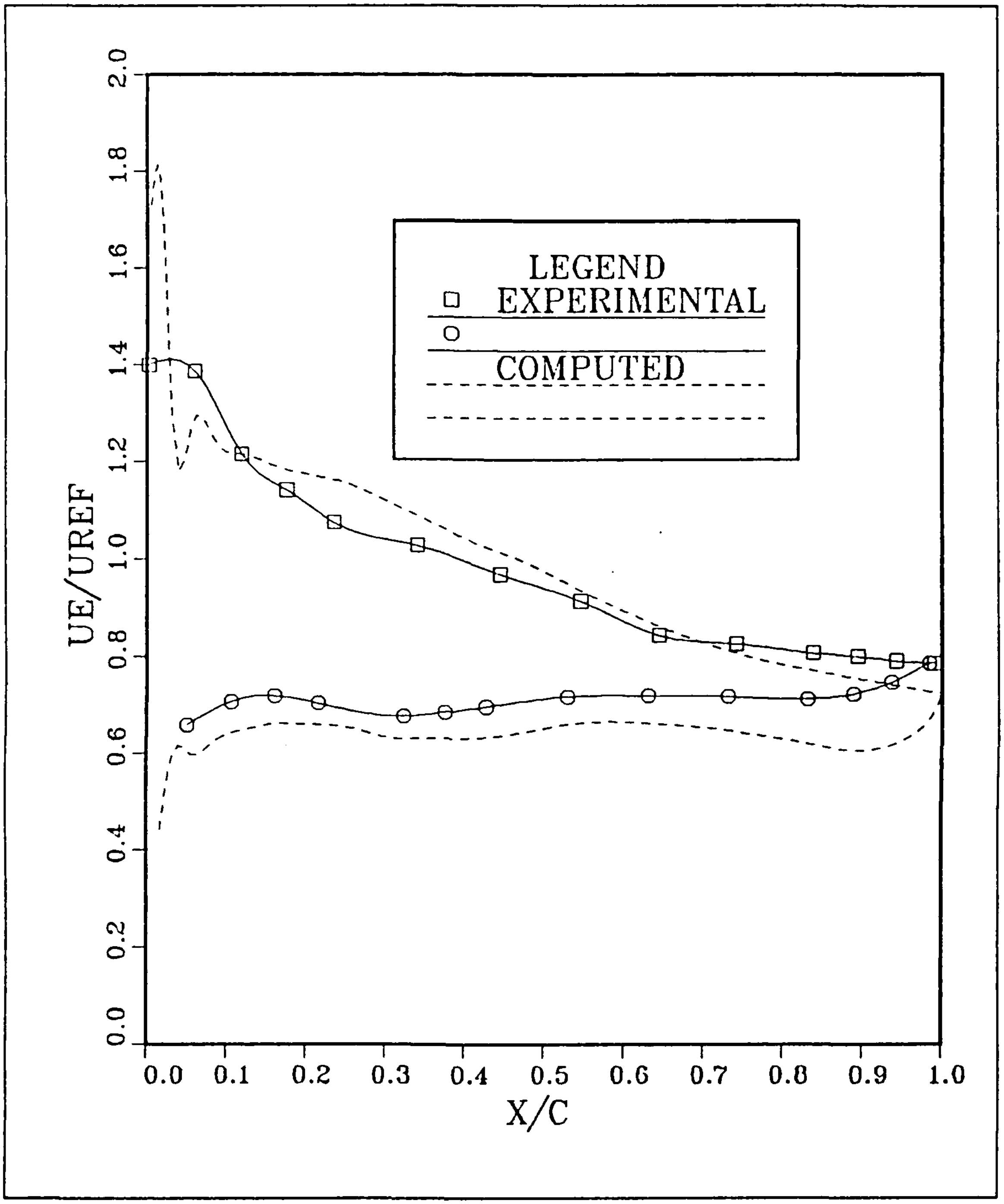

Figure 12. External velocity at $\beta = 46°$

3. Boundary Layer Thickness

The boundary layer thickness as computed by the code was compared with the experimental results by comparing the displacement thicknesses.

It was found that on the lower surface the computed and the actual displacement thickness agree quite well, as can be seen in Figure 13 on page 35 for $\beta = 40°$, Figure 14 on page 36 for $\beta = 43.4°$ and in Figure 15 on page 37 for $\beta = 46°$.

On the upper surface the displacement thicknesses computed by the program are significantly thinner than those measured experimentally. The difference between the computed and the actual thickness increases along the blade and it increases with increased inlet angle. It was found that by using a different expression for the inner region eddy viscosity (as mentioned in chapter II), the displacement thickness can be increased, but the difference between the actual and the computed thickness is still substantial, especially at the higher inlet angles. Figure 16 on page 38, Figure 17 on page 39 and Figure 18 on page 40 shows the displacement thickness on the upper surface for the three inlet angles, with the original and the modified eddy viscosity models.

The large error in the prediction of the boundary layer thickness, can be the result of several reasons:

1. The transition model used in the code, sets the onset of transition at the first point of laminar separation. It causes rapid transition to turbulent flow which reattaches immediately, resulting in a very small separation bubble compared to the bubble observed in the experiment.
2. The turbulent model used in the code could be inaccurate. It was derived based on empirical data obtained in single airfoil experiments and not with cascades. In addition the present model does not include the effects of the free stream turbulence (that was relatively high in the experiment).
3. The boundary layer as measured in the experiment was quite thick, especially at the higher inlet angles (it reached 15% of the chord at $\beta = 46°$). Such a thick boundary layer may violate the basic assumptions on which the boundary layer equations, and the interaction law, were based (especially when the spacing between the blades is small, 60% chord in this case).

It was suggested that one of the possible reasons to the inaccurate prediction of the boundary layer is the blunt trailing edge of the blade, that might cause difficulties in the computations. A modified blade, with a sharp trailing edge has been run, and the displacement thickness distribution can be seen in Figure 19 on page 41. As can be seen in the figure the sharp trailing edge affects only the boundary layer adjacent to the trailing edge, and therefore cannot provide an explanation to the difference between the actual and the computed displacement thickness.

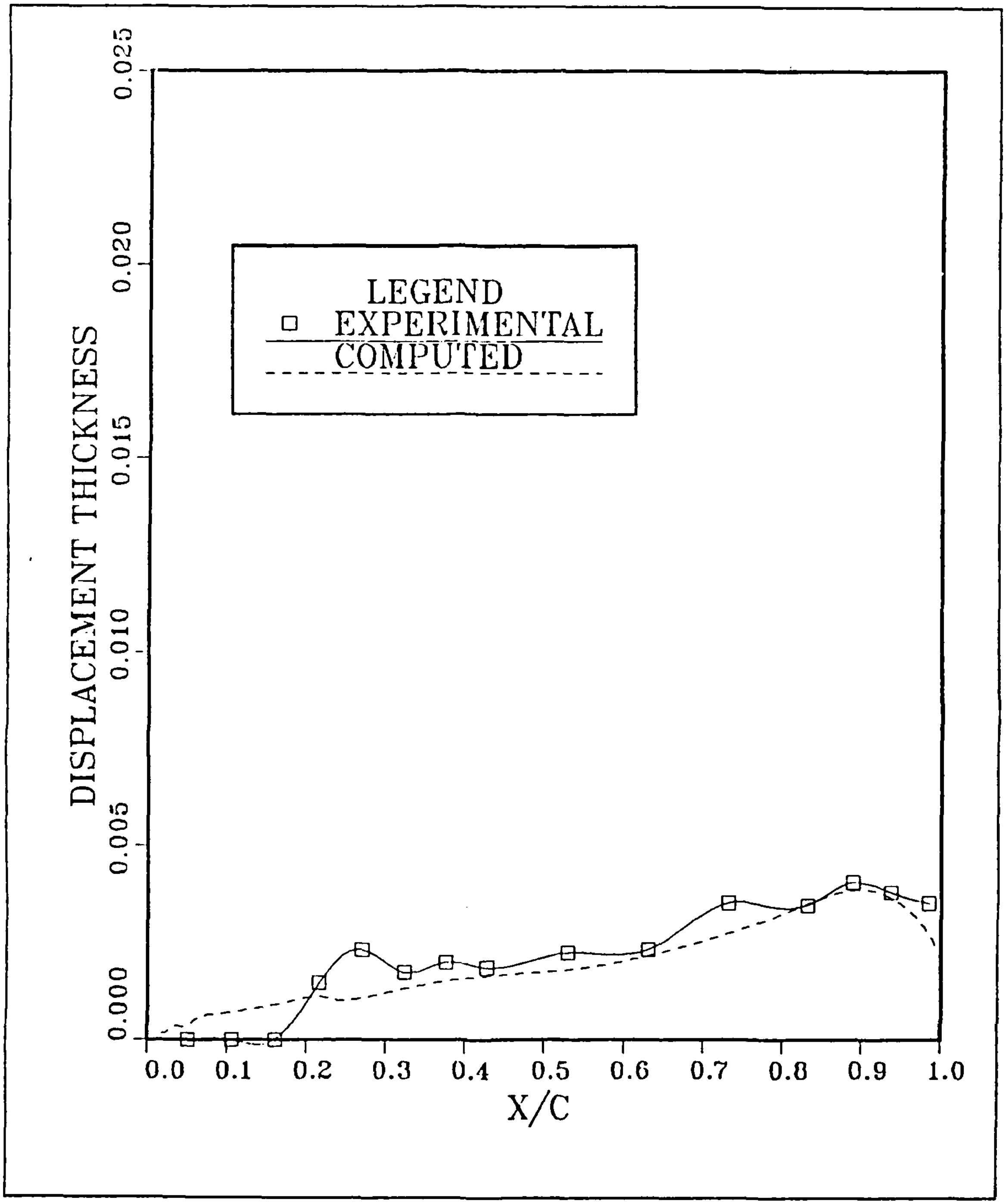

Figure 13. Displacement thickness on the lower surface (β = 40°)

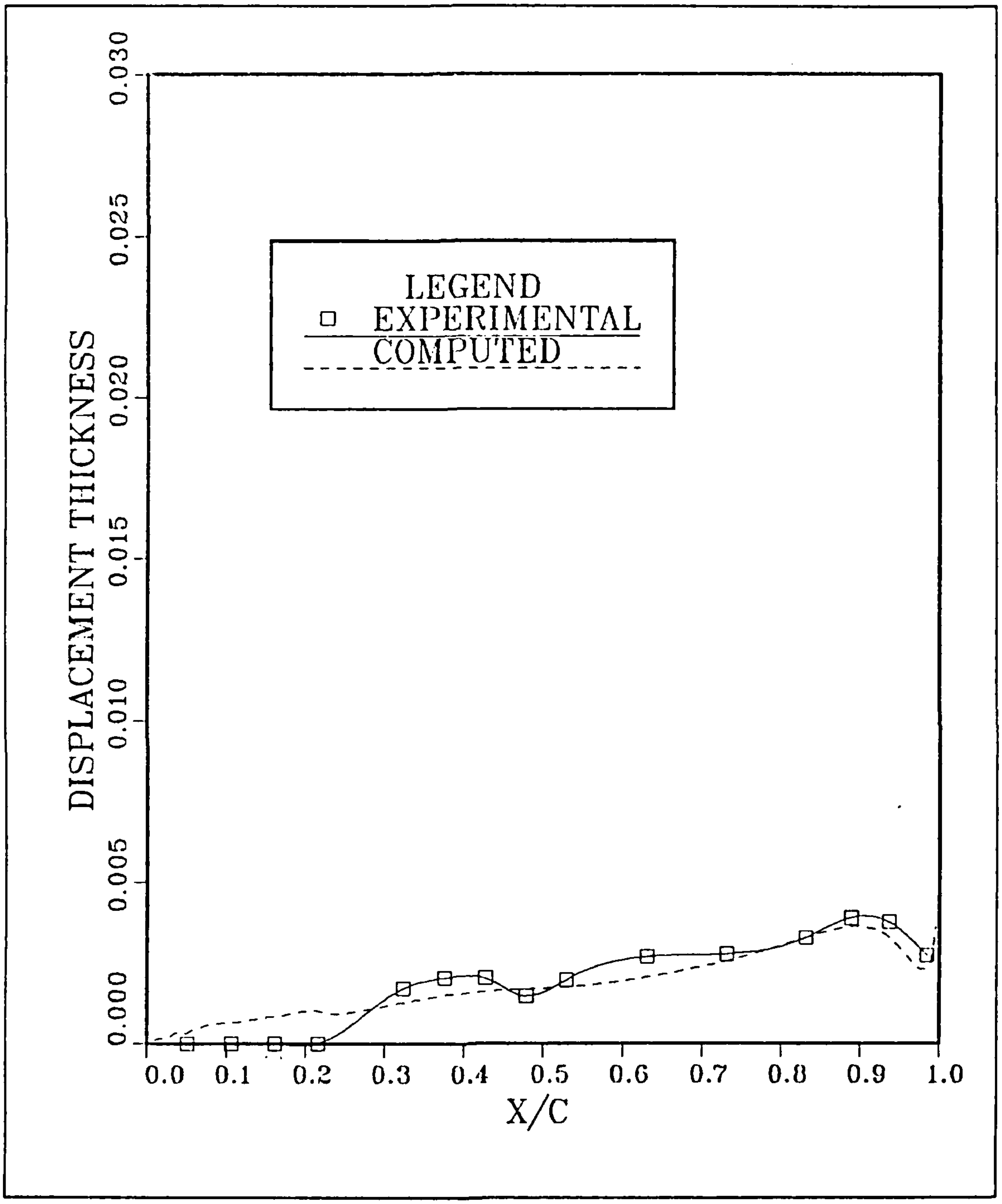

Figure 14. Displacement thickness on the lower surface (β = 43.4°)

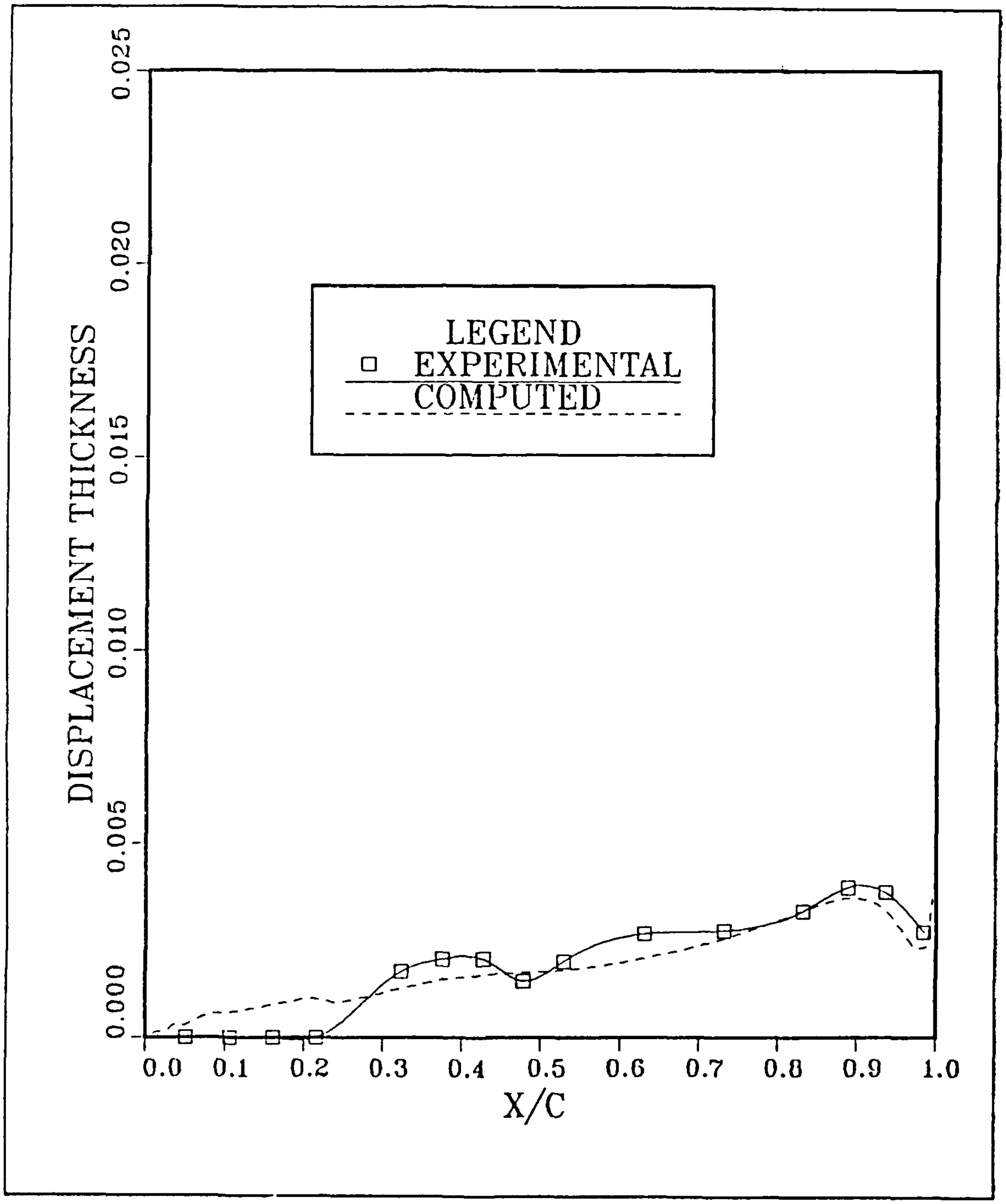

Figure 15. Displacement thickness on the lower surface (β = 46°)

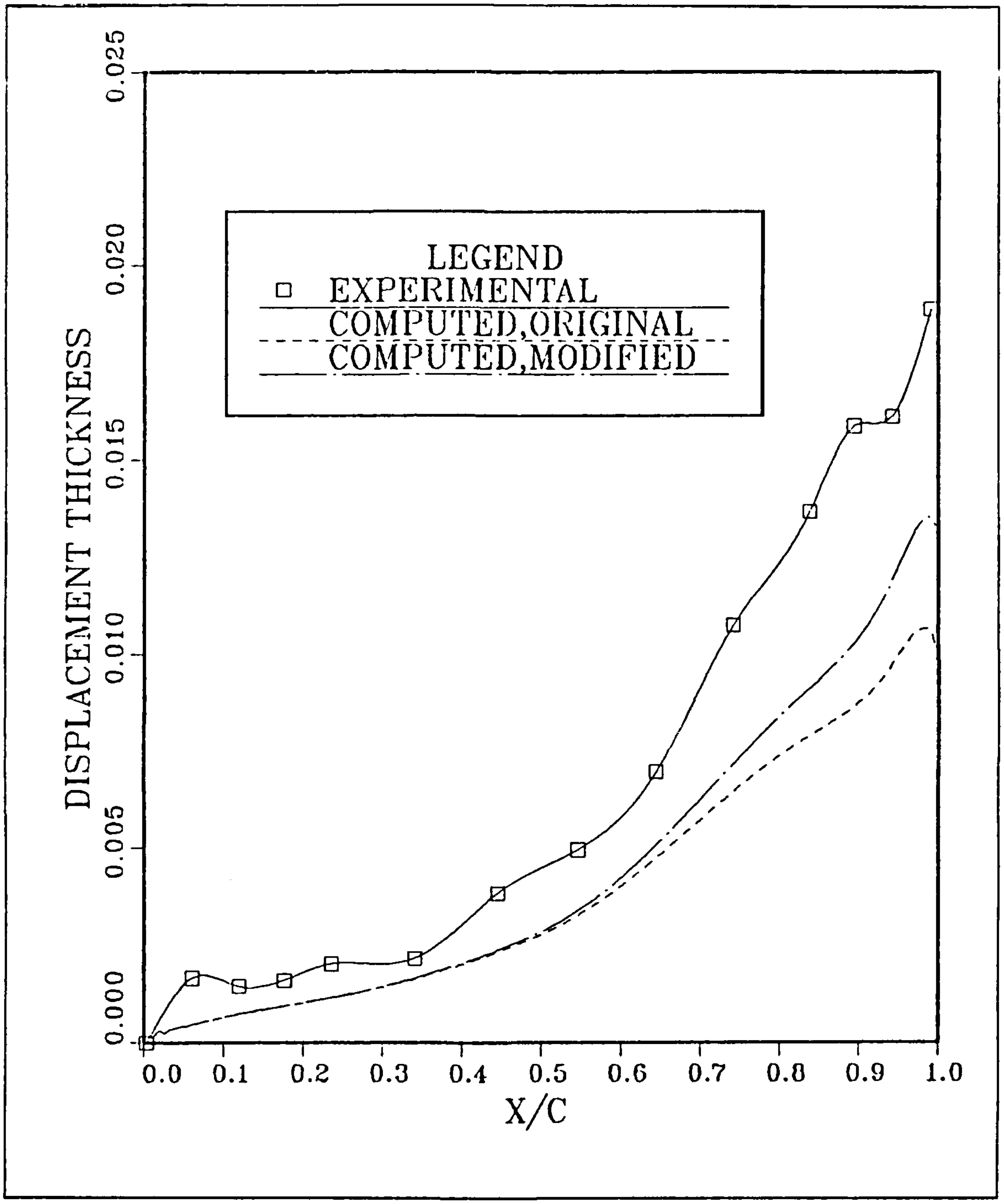

Figure 16. Displacement thickness on the upper surface (β = 40°)

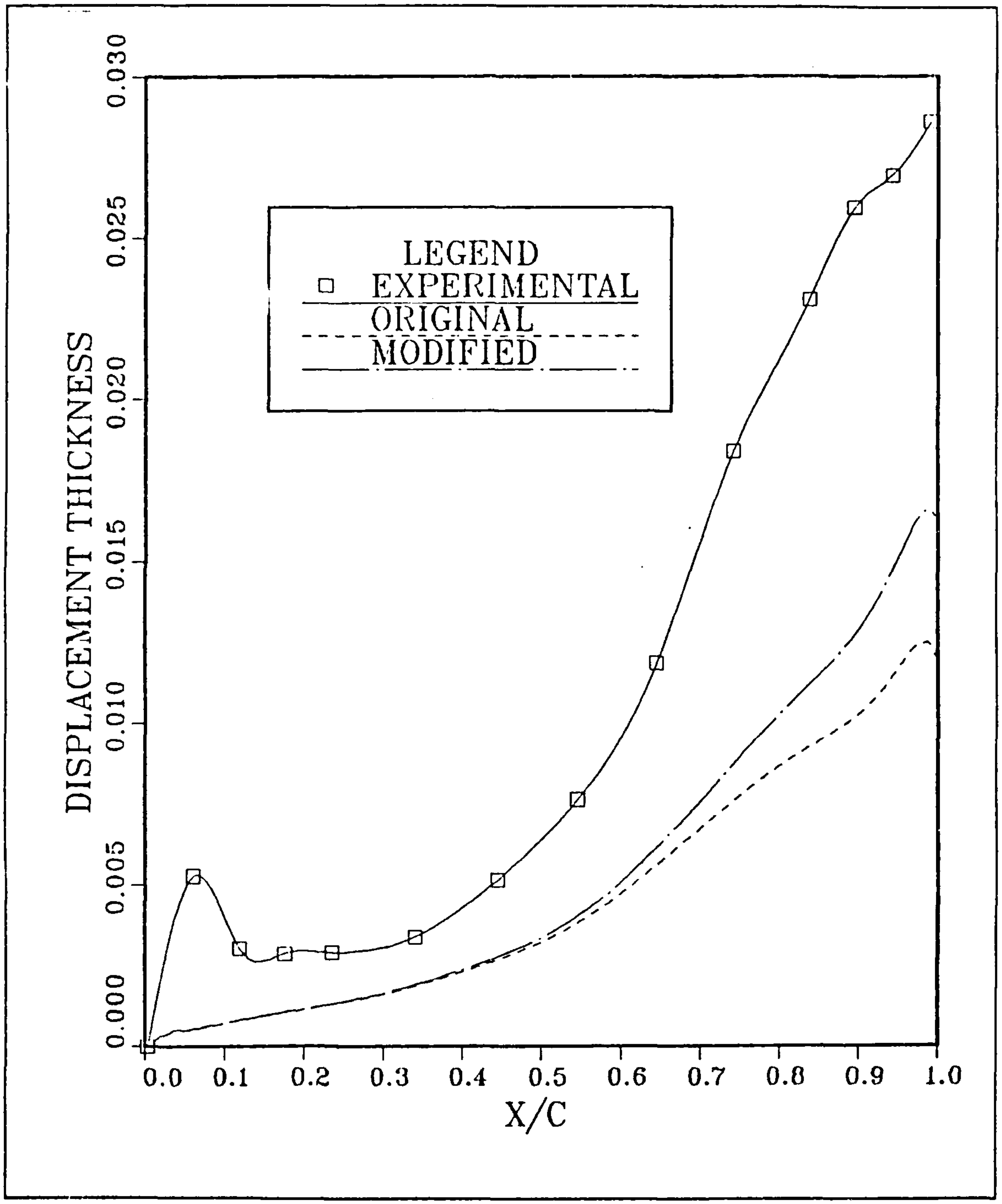

Figure 17. Displacement thickness on the upper surface (β = 43.4°)

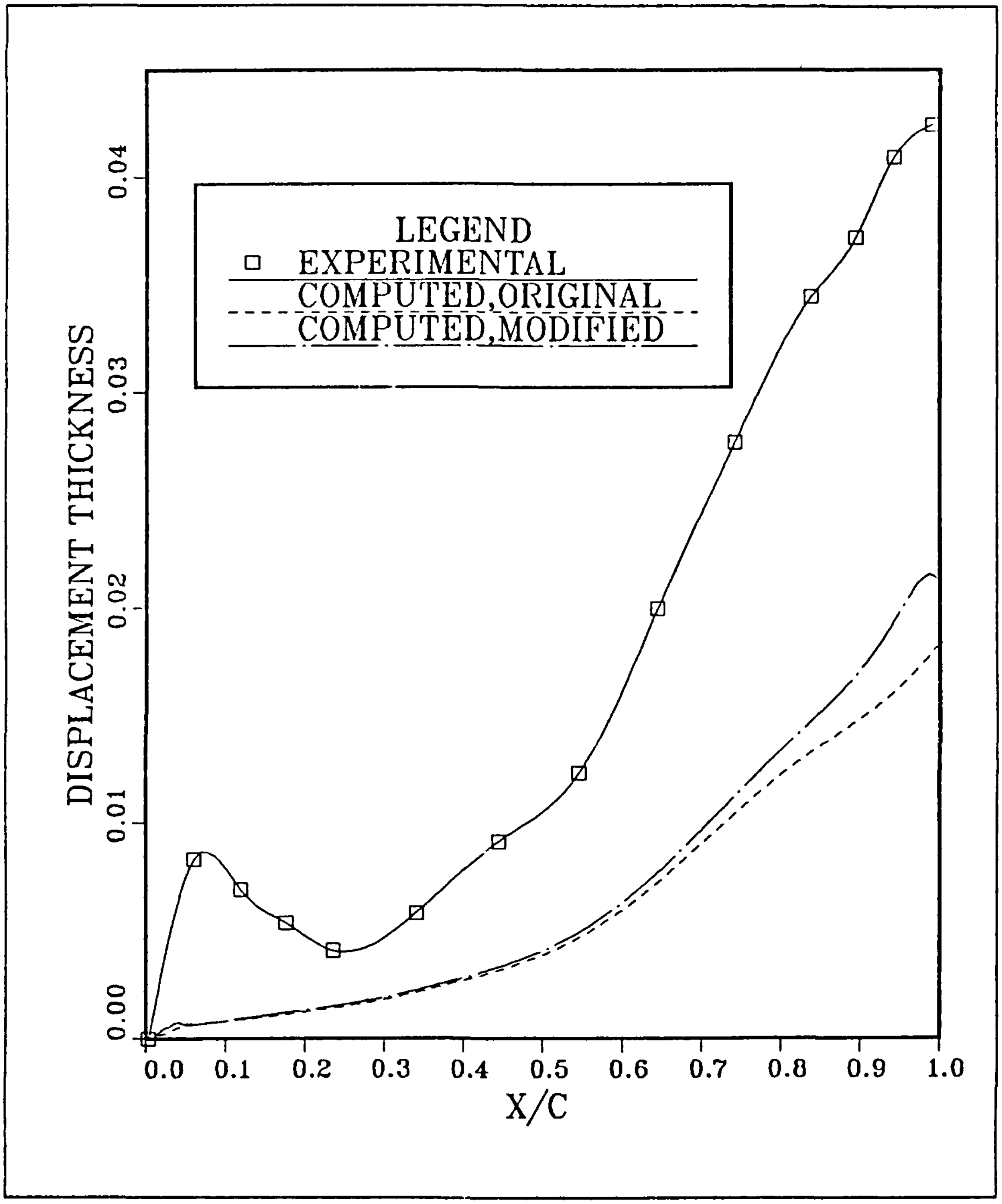

Figure 18. Displacement thickness on the upper surface (β = 46°)

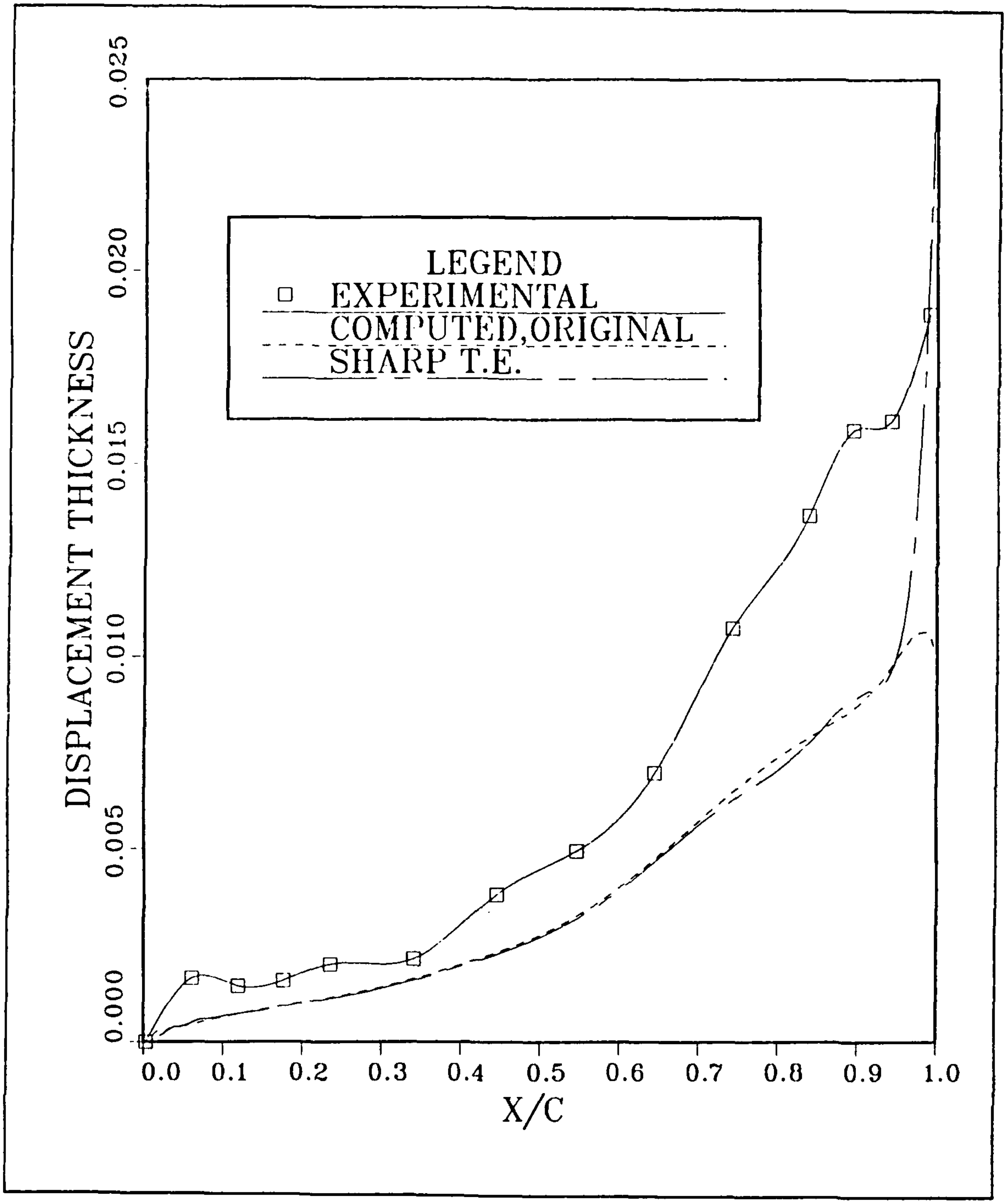

Figure 19. The effect of sharp trailing edge.

4. Comparison to a Navier Stokes Code

A limited comparison of the experimental and the computed results with a Navier Stokes (N.S.) code calculations has been performed. The N.S. code has been developed and run by S. J. Shamroth of Scientific Research Associates inc. in cooperation with Pratt and Whitney Aircraft.

Since the N.S. code does not compute the displacement thickness, the velocity profiles near the surface of the blade were compared. The comparisons were made at 90% chord on the suction surface for all three inlet angles.

At the design point, $\beta = 40°$, shown in Figure 20 on page 43, both the interactive code and the N.S. code failed to predict accurately the actual velocity profile. In this case the interactive code seems to yield somewhat better results than the N.S. code.

At the higher inlet angles, $\beta = 43.4°$ and $\beta = 46°$, shown in Figure 21 on page 44 and in Figure 22 on page 45 respectively, the N.S. calculations show significantly better agreement with the experimental results than the interactive code.

From these comparisons, it can be seen that the interactive code deviation from the actual results increases with increased inlet angle (increased loading of the cascade), whereas the N.S. code deviation seems to decrease with increased inlet angle.

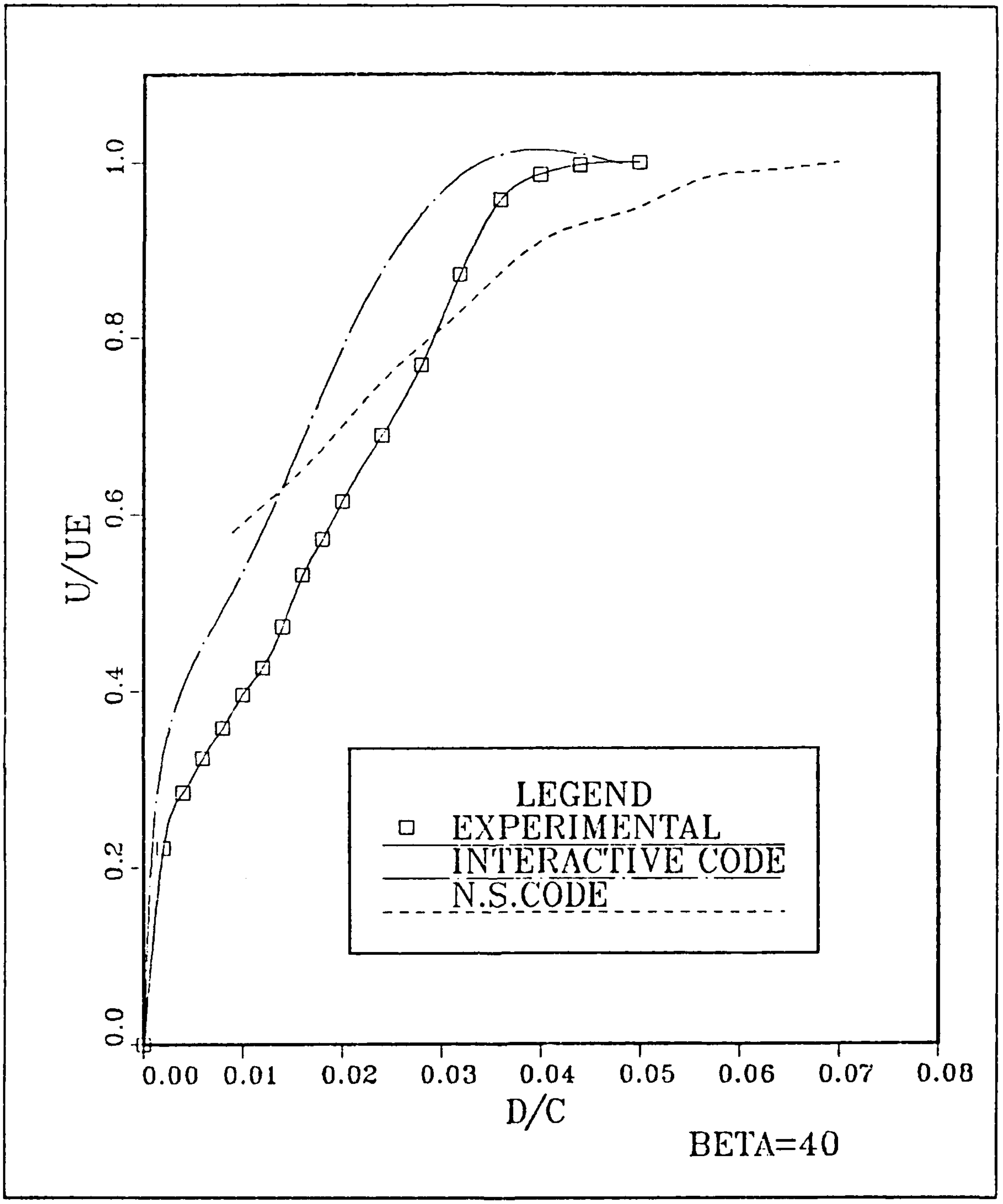

Figure 20. The results of the N. S. code at $\beta = 40°$

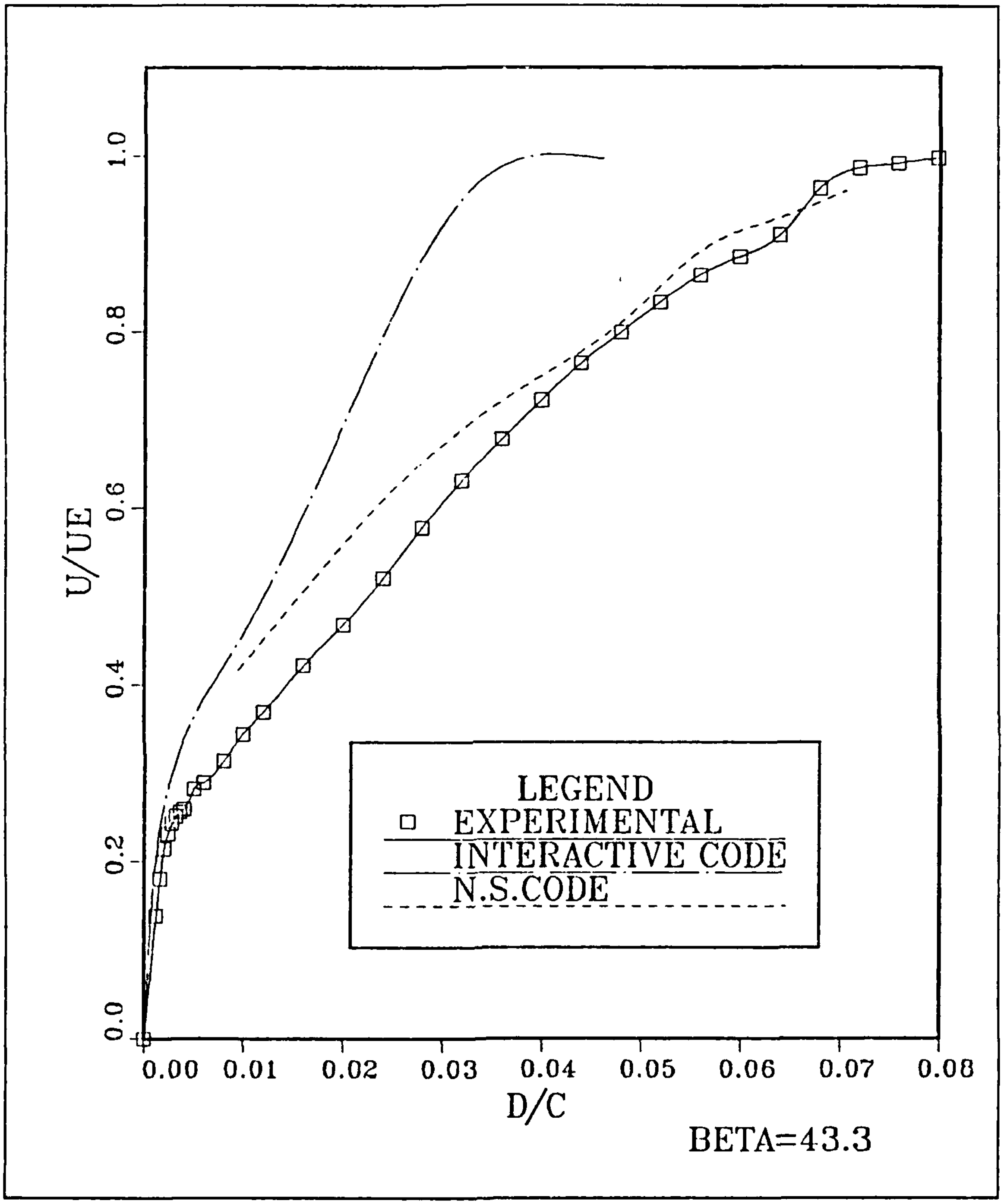

Figure 21. The results of the N. S. code at $\beta = 43.4°$

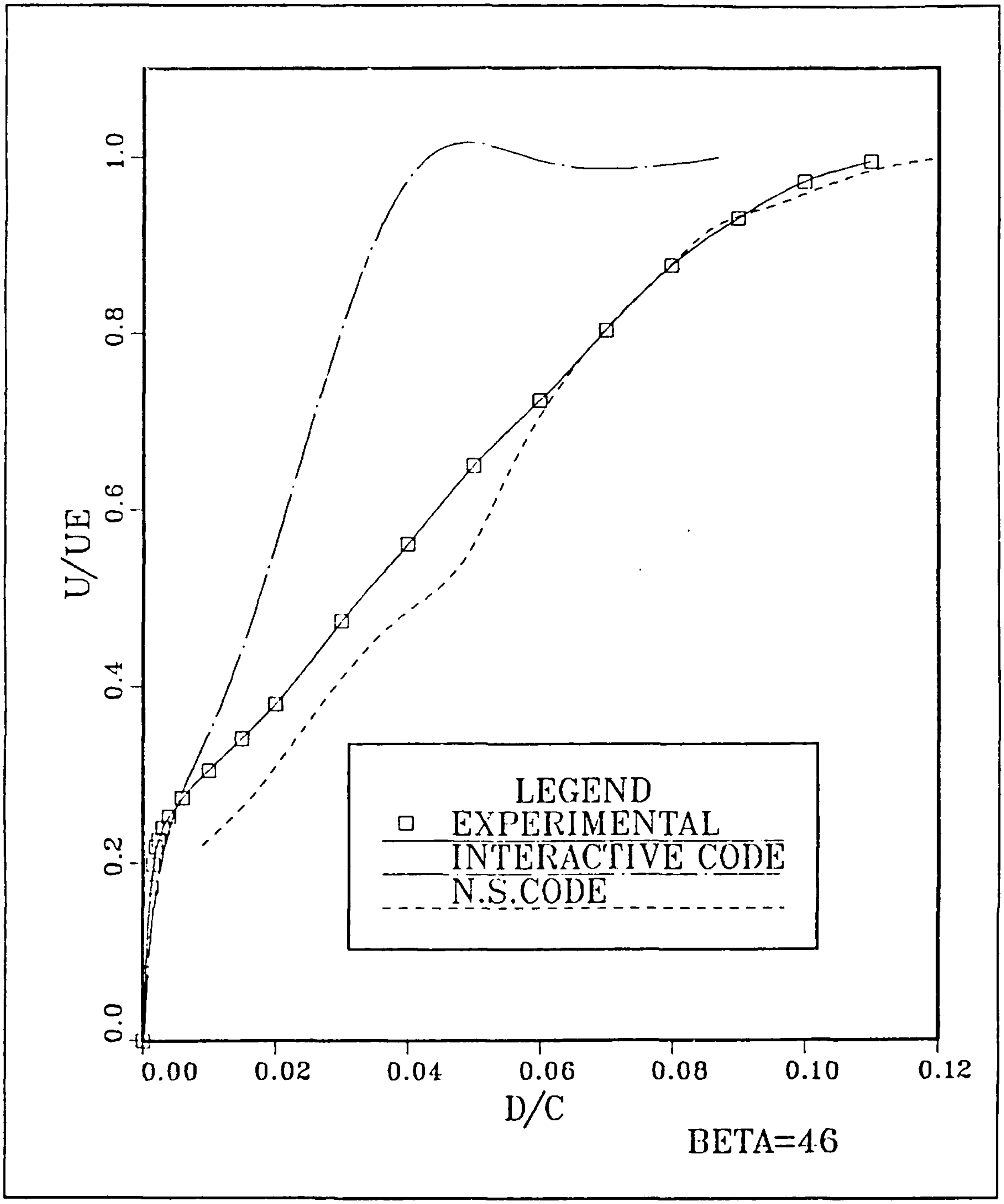

Figure 22. The results of the N. S. code at $\beta = 46°$

B. P & W CASCADE

The experimental data for the P & W cascade was obtained at inlet flow angle of 52°, at $M = 0.11$, and Reynolds number of 478000. The cascade had a stagger angle of 15.75° and 0.7 spacing. A general layout of the cascade is shown in Figure 23 on page 47.

A comparison of the computed and the measured pressure coefficients on the blade is shown in Figure 24 on page 48. There is a good agreement between the computed and the measured C_p.

The displacement thickness was measured in the experiment only at 96.8% of chord. This measurement is compared to the computed results in Figure 25 on page 49. As can be seen, the computed and the measured data agree almost perfectly on the lower surface, and quite well on the upper surface. The difference observed on the upper surface is caused by the early prediction, by the code, of trailing edge separation, a short distance upstream of the actual location. This can also be observed when comparing the velocity profiles at that point, in Figure 26 on page 50. The computed velocity curve shows a small zone of reversed flow near the surface of the blade. This reversed flow could be the result of a too early prediction of trailing edge separation by the code, or it could have existed in the actual flow but not detected because of its size.

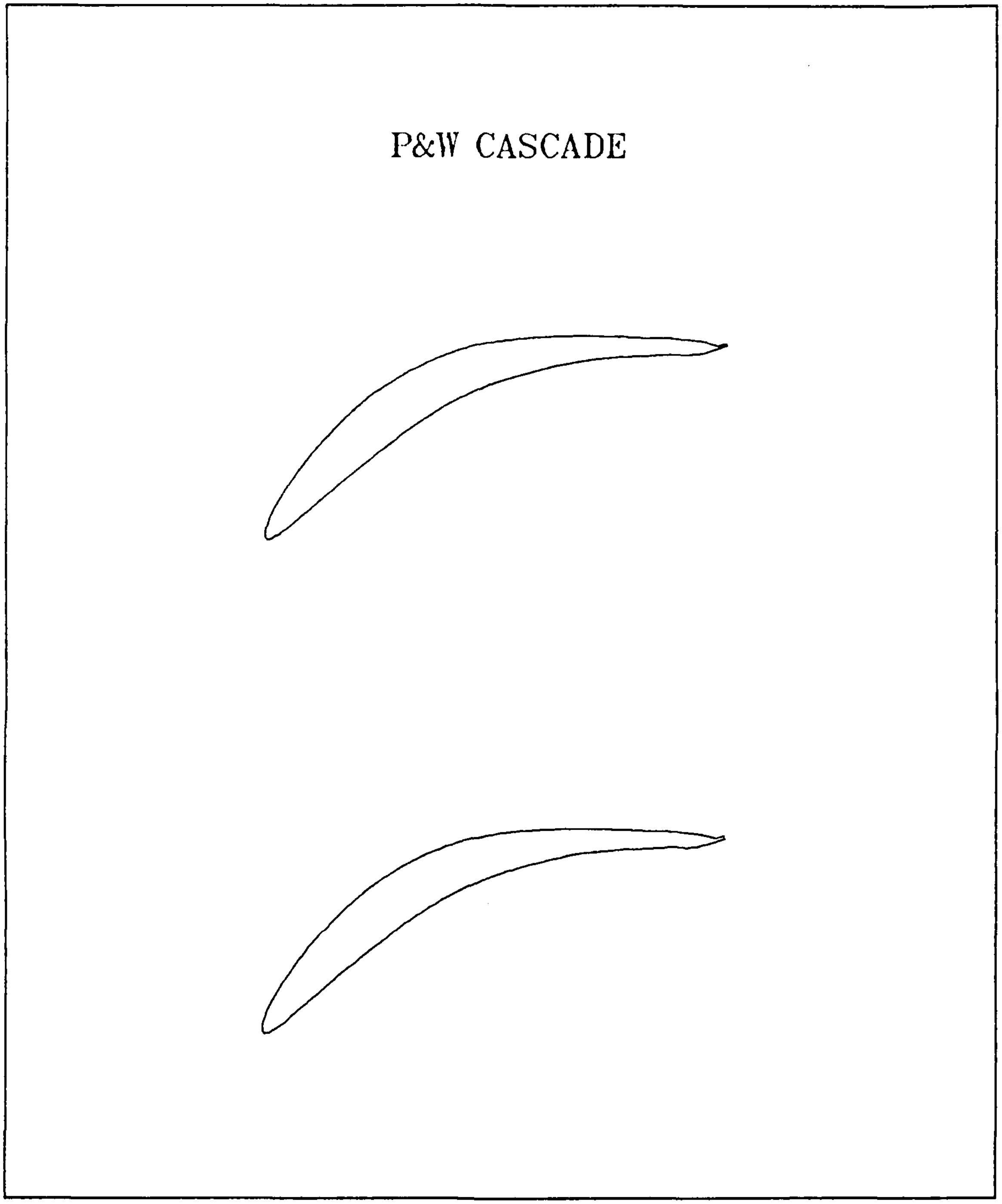

Figure 23. Pratt & Whiteny cascade

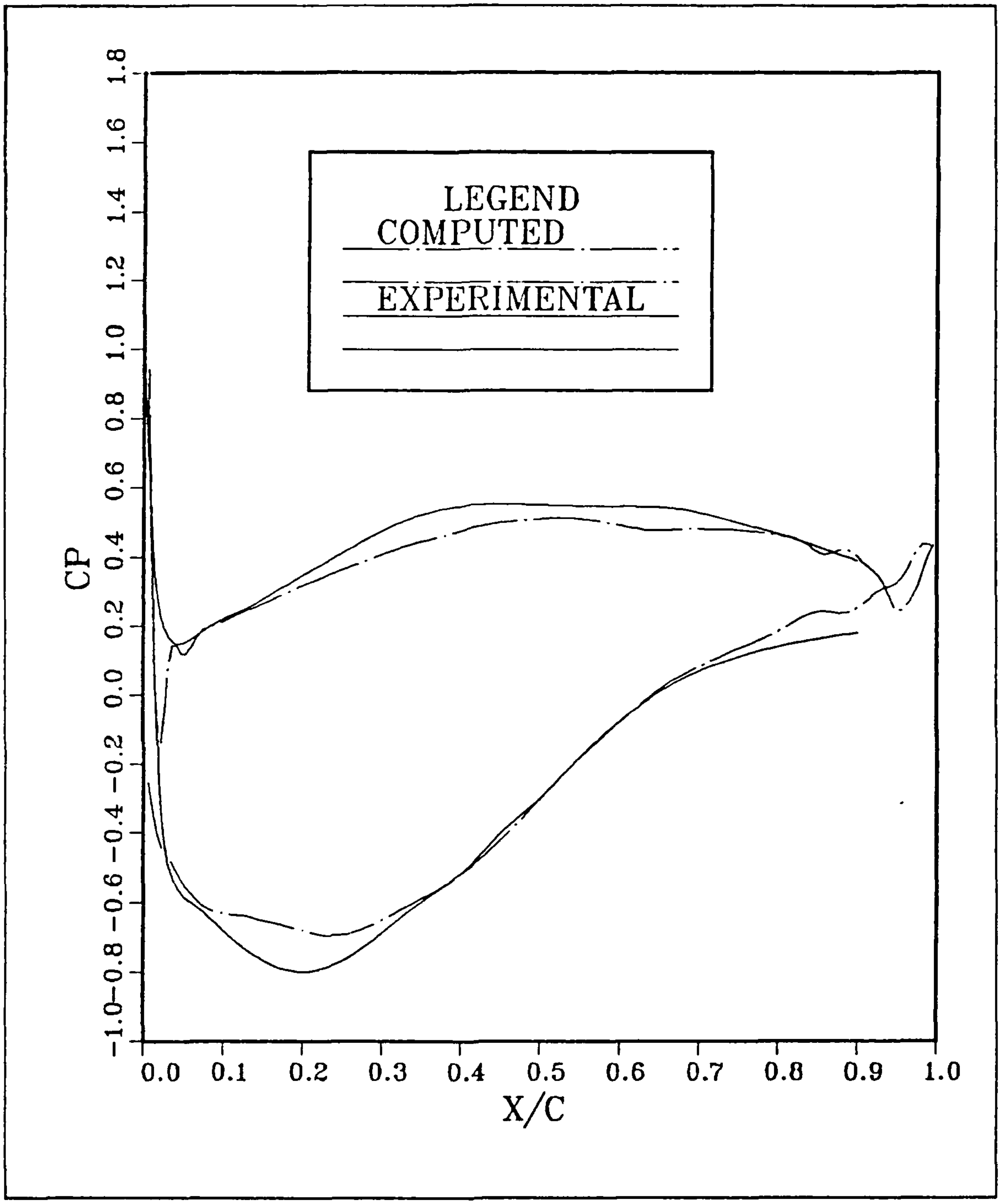

Figure 24. Comparison of pressure coefficient.

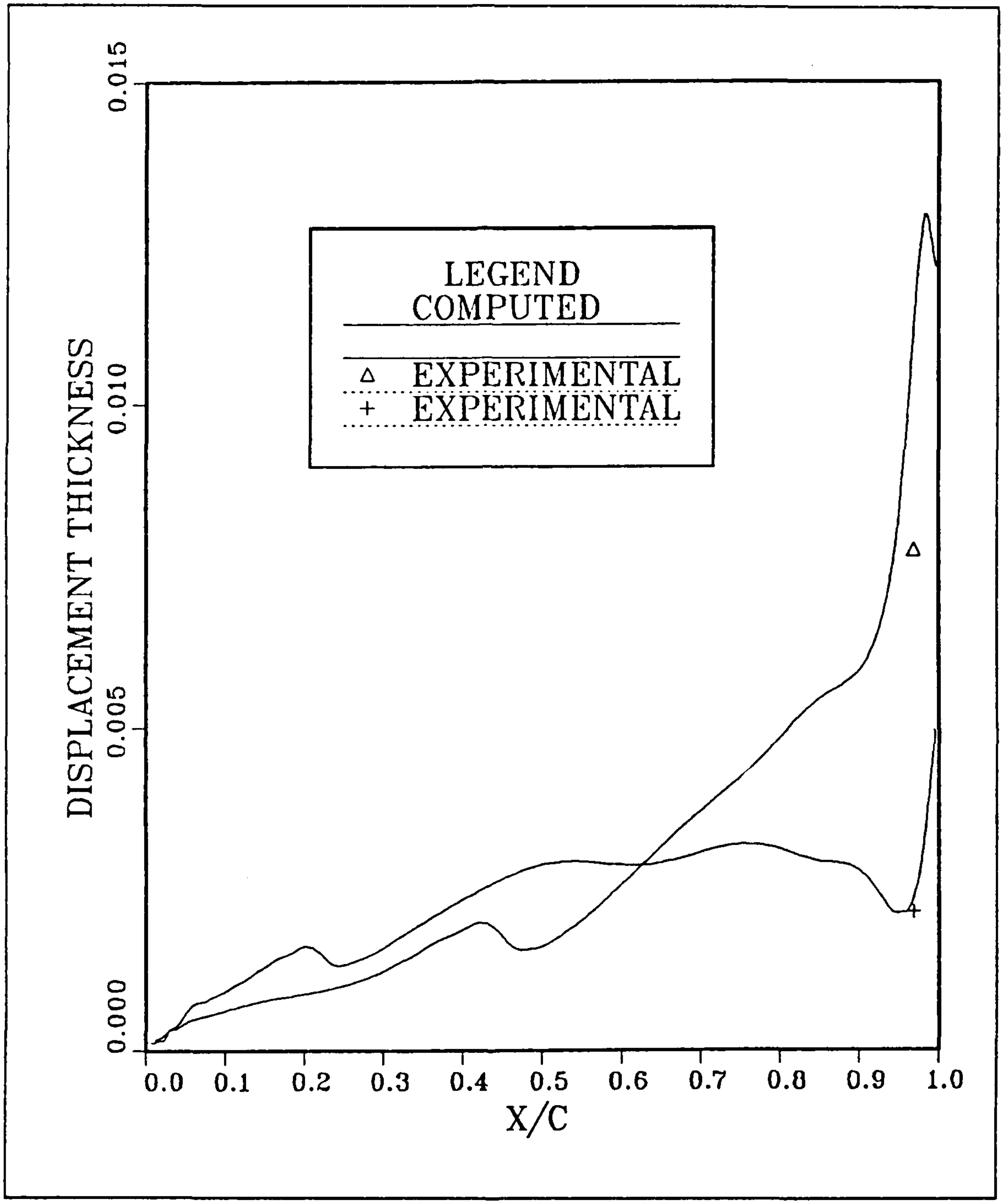

Figure 25. Displacement thickness comparison: Experimental data shown at 96.8% chord.

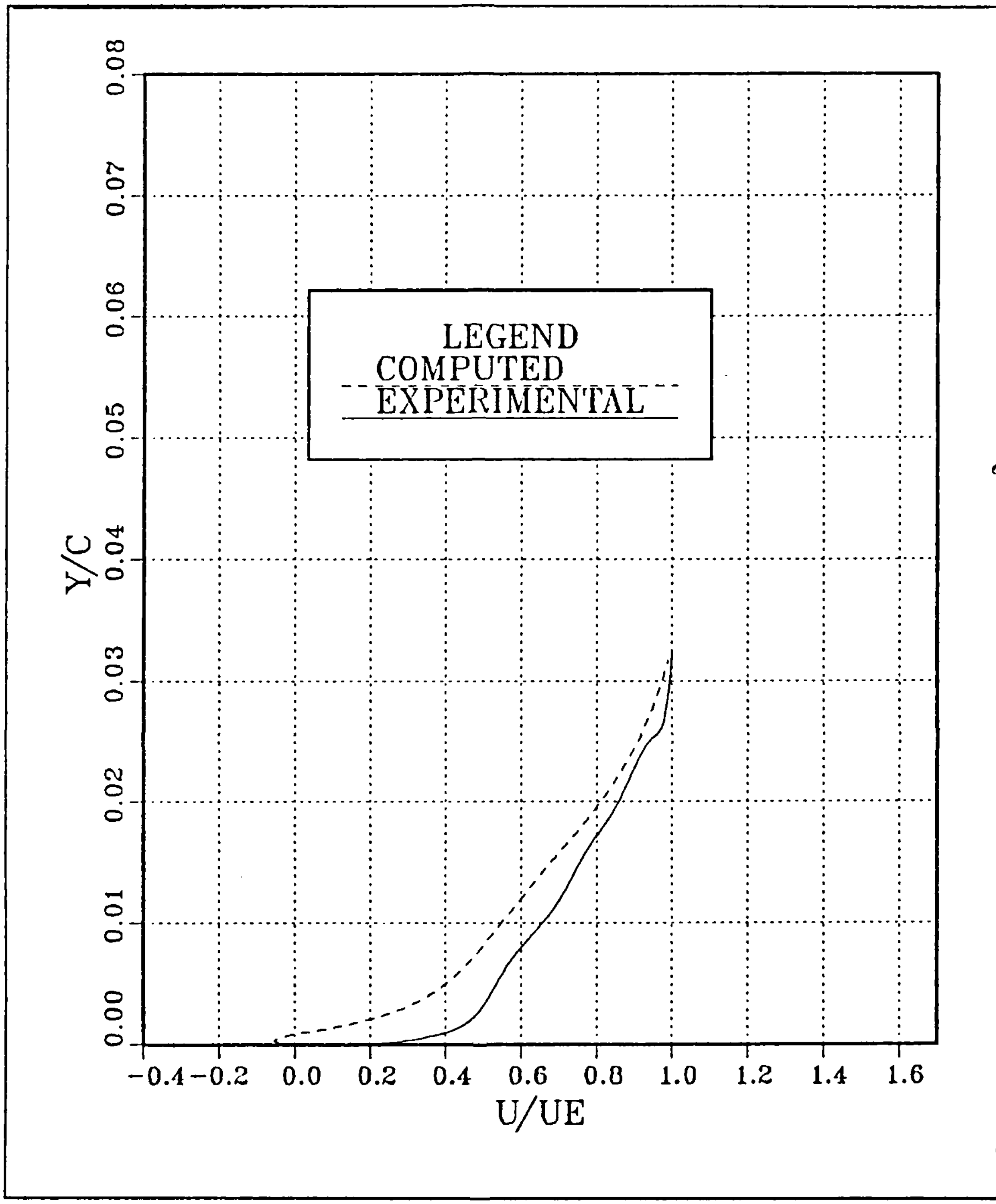

Figure 26. Velocity profile at 96.8% chord on the upper surface.

C. C4 CASCADE

The C4 cascade has a stagger angle of 29.5°, a camber angle of 31.1° and spacing of 0.992. It has been tested at Reynolds numbers of about 200000, and inlet angles of 34.1° to 47.7° (which corresponds to incidence angles of -10.9° to 2.7°). The general layout of the cascade is shown in Figure 27 on page 52. A computer code that generates the coordinates of the blade and a summary of some experimental results are given in Appendix B.

The code was run with the intermittency constant $G_\gamma = 10$. Higher values of G_γ (above 100) caused numerical problems in the code. The onset of transition was first taken at the point where it was observed in the experiment. At the lower inlet angle it seems that a better agreement with the experimental results can be obtained by delaying the onset of transition but trying to implement it resulted in numerical breakdown of the computation. At the higher inlet angles, better agreement with the experimental results was achieved by initiating the transition earlier (at 26% chord for $\beta = 45.6°$ and at 21% for $\beta = 47.7°$ as compared to 44% and 36% chord as observed in the experiment).

1. Displacement Thickness

Comparisons of the experimental data to the computed displacement thickness are shown in Figure 28 on page 53 for inlet angle of 34.1°, in Figure 29 on page 54 for inlet angle of 36.3°, in Figure 30 on page 55 for inlet angle of 45.6° and in Figure 31 on page 56 for inlet angle of 47.7°.

As can be seen in the figures, there is a good agreement between the actual and the computed results at the two lower angles ($\beta = 34.1°$ and $\beta = 36.3°$, in which the incidence angles were negative). At the two higher angles, $\beta = 45.6°$ and $\beta = 47.7°$ the computed results agree with the actual results up to about 70% chord, and then the displacement thickness predicted by the code becomes much thicker than the actual one.

The code predicted a large flow separation area starting at about 70% chord at the lower inlet angles, and at about 46% chord at the higher inlet angles. This flow separation was not observed in the experiment. The discrepancies between the computed and the actual results behind 60% to 70% chord can be explained by the inaccurate calculations by the code due to the large separated areas. When the code encounters separation, several approximations are made (like the FLARE approximation) based on the assumption that the separated area is small. When the separated area is large, these approximations may result in inaccurate prediction of the flow field.

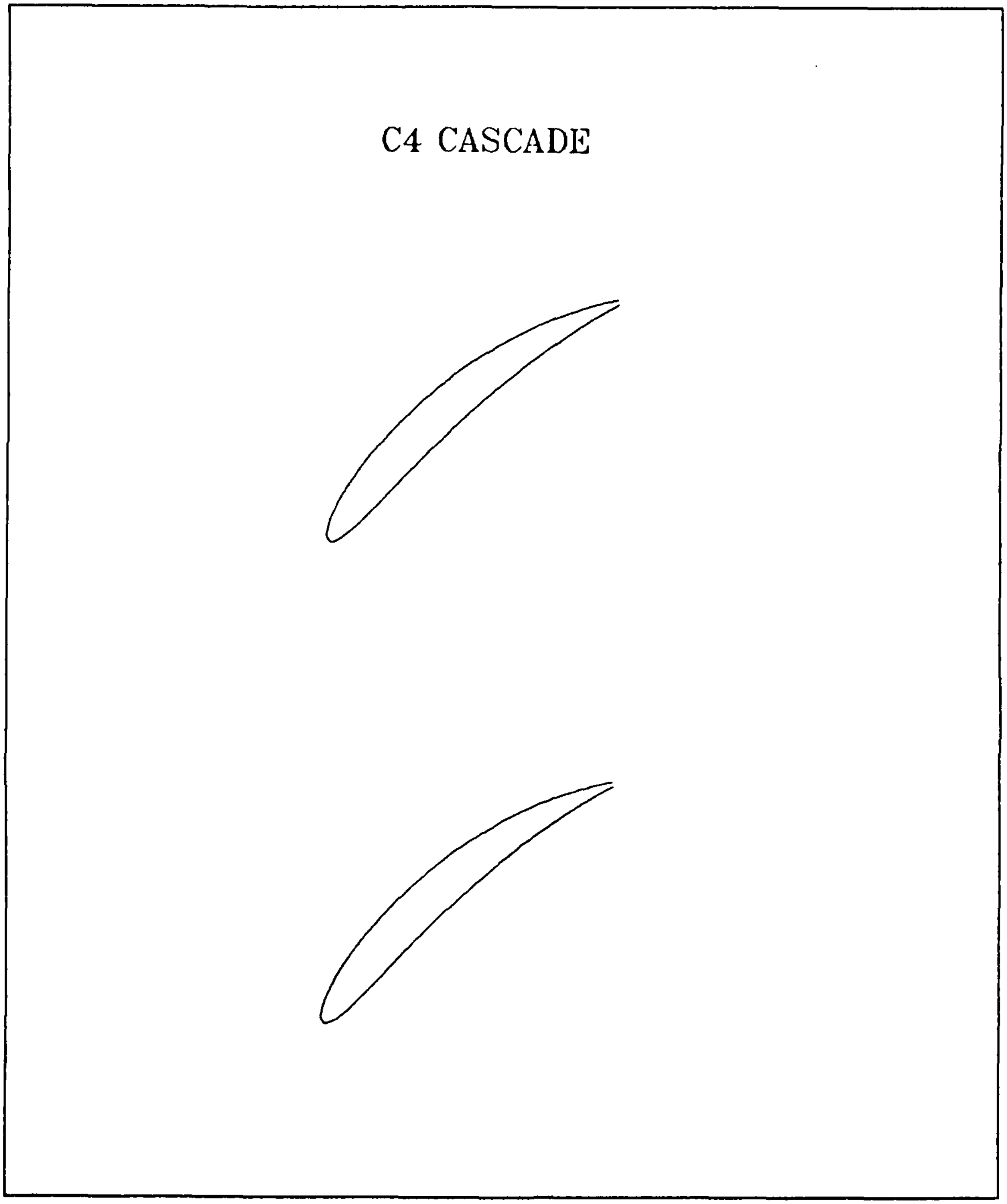

Figure 27. C4 Cascade

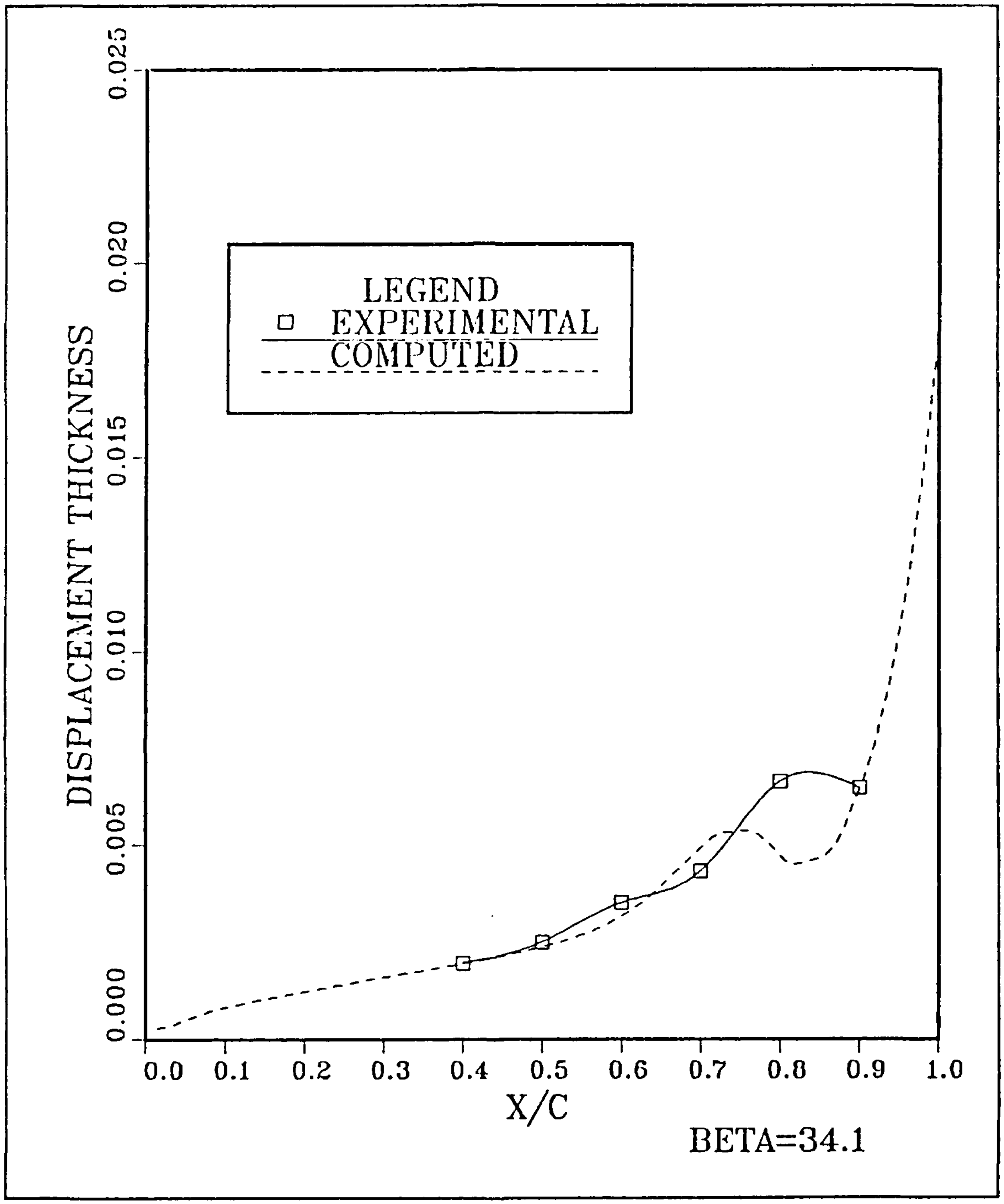

Figure 28. C4 cascade at β = 34.1°: Displacement thickness comparison with computed results.

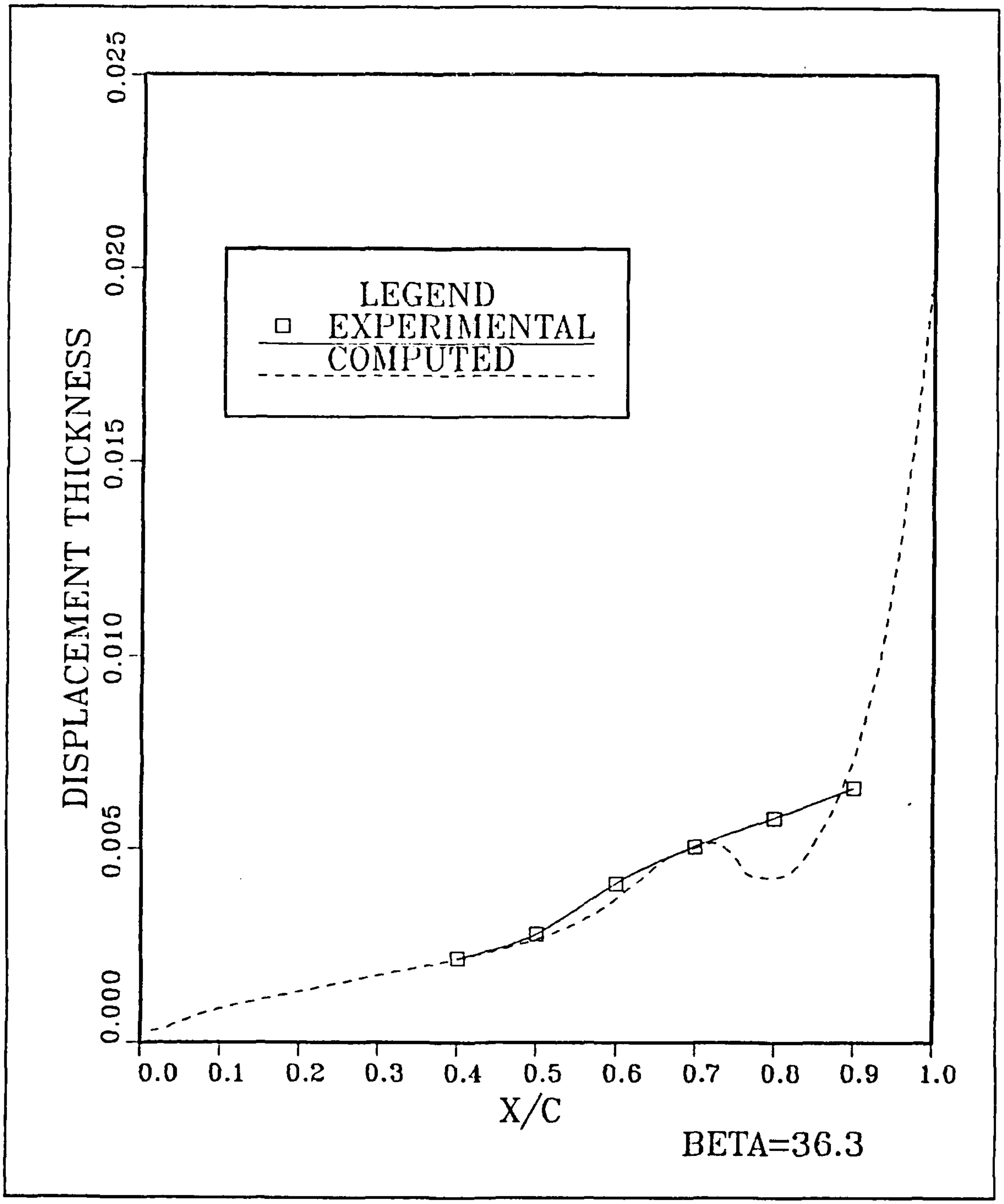

Figure 29. **C4 cascade at β = 36.3°:** Displacement thickness comparison with computed results.

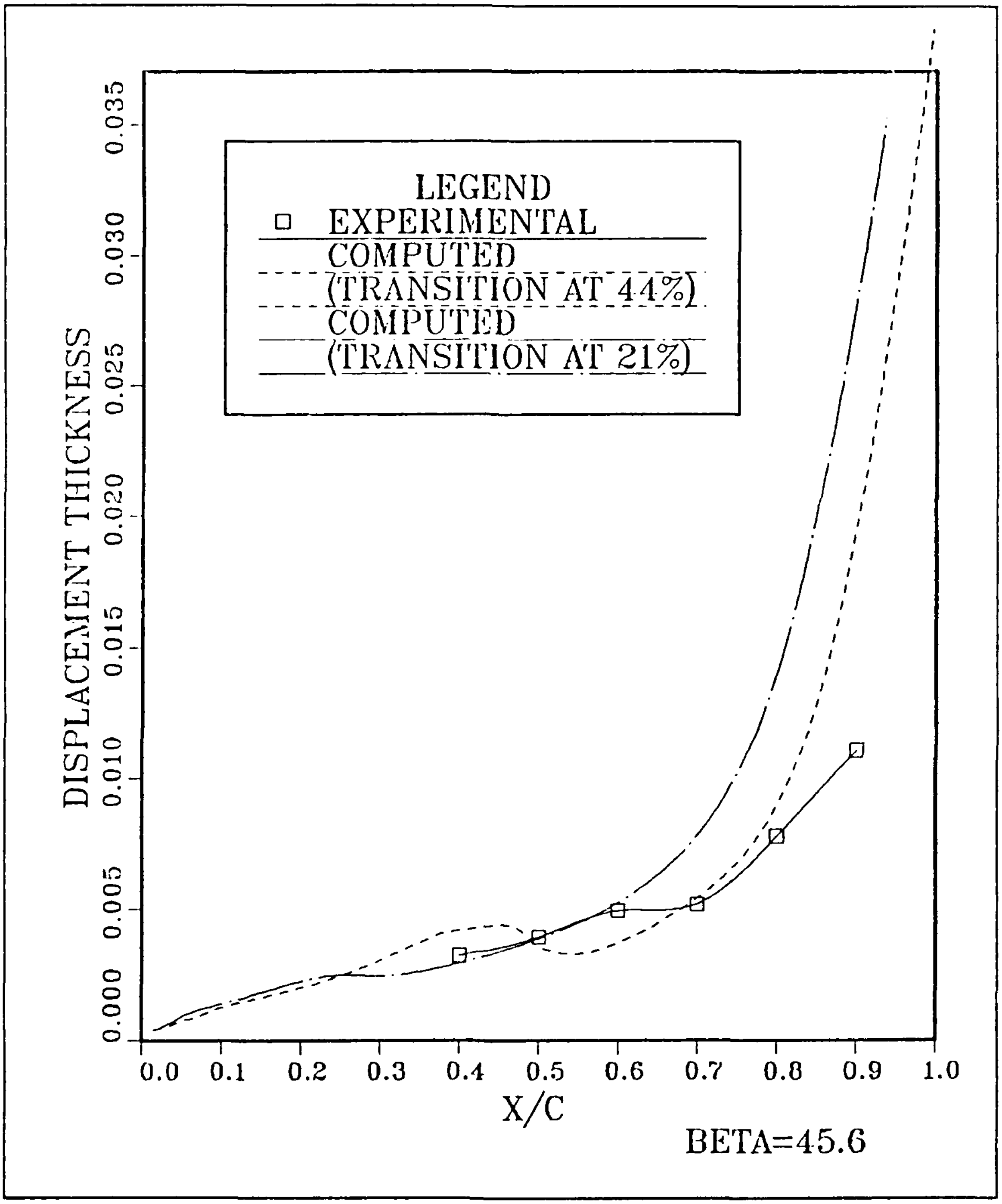

Figure 30. C4 cascade at β = 45.6°: Displacement thickness comparison with computed results (G= 10).

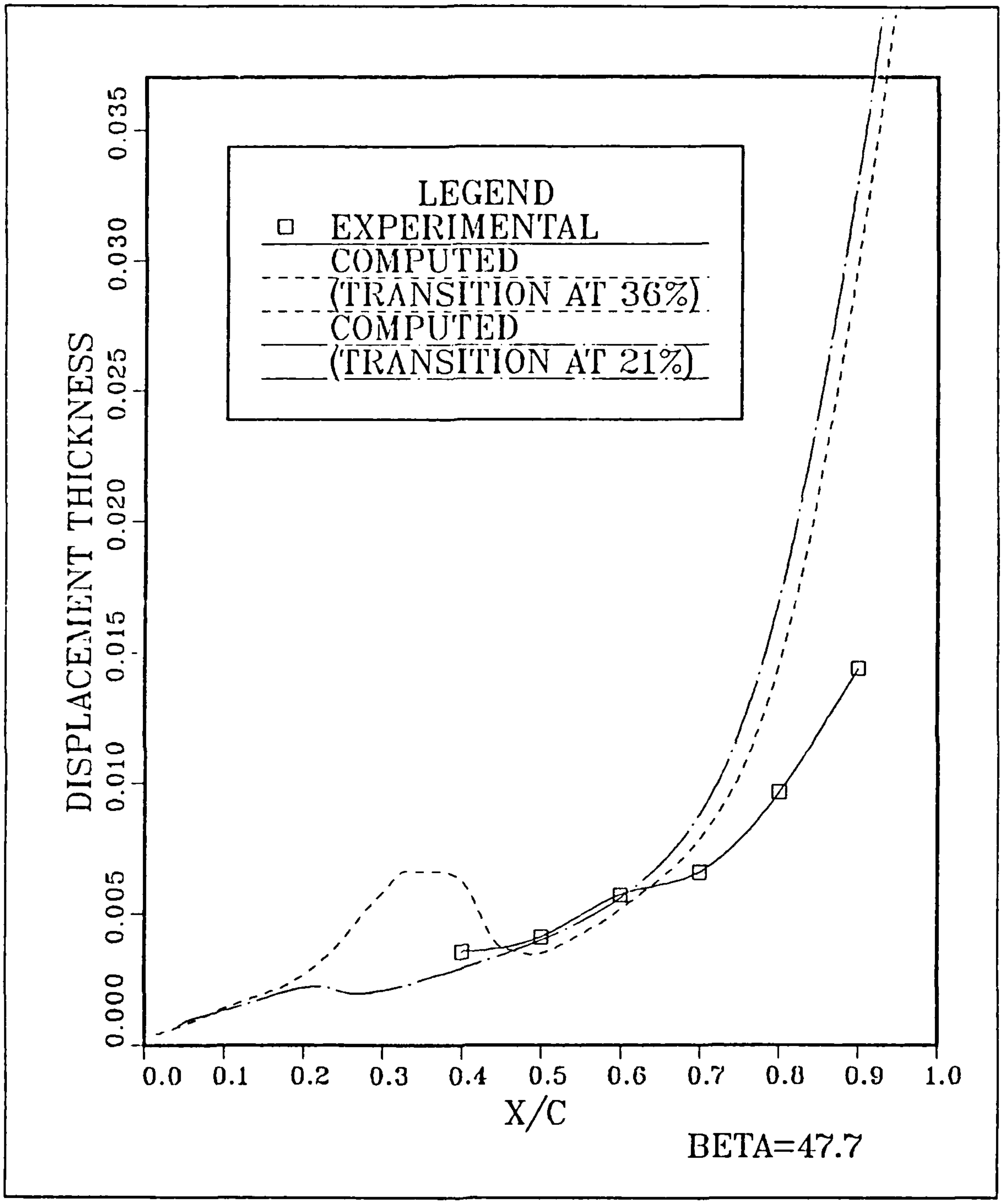

Figure 31. **C4 cascade at β = 47.7°:** Displacement thickness comparison with computed results (G = 10).

2. External Velocity and Velocity Profiles Comparisons

A comparison of the external velocity on the upper surface of the blade is shown in Figure 32 on page 58 for inlet angle of 45.6° and in Figure 33 on page 59 for inlet angle of 47.7°. It can be seen that there is a good agreement between the experimental and the computed results up to about 80% chord. Near the trailing edge the computed results deviate from the experimental results due to the inaccuracy in the calculations of the displacement thickness.

A comparison of the velocity profiles in the boundary layer at 50% chord is shown in Figure 34 on page 60 for inlet angle of 34.1° and in Figure 35 on page 61 for inlet angle of 36.3°. The agreement between the calculated velocity profiles and the measured velocity profiles is very good.

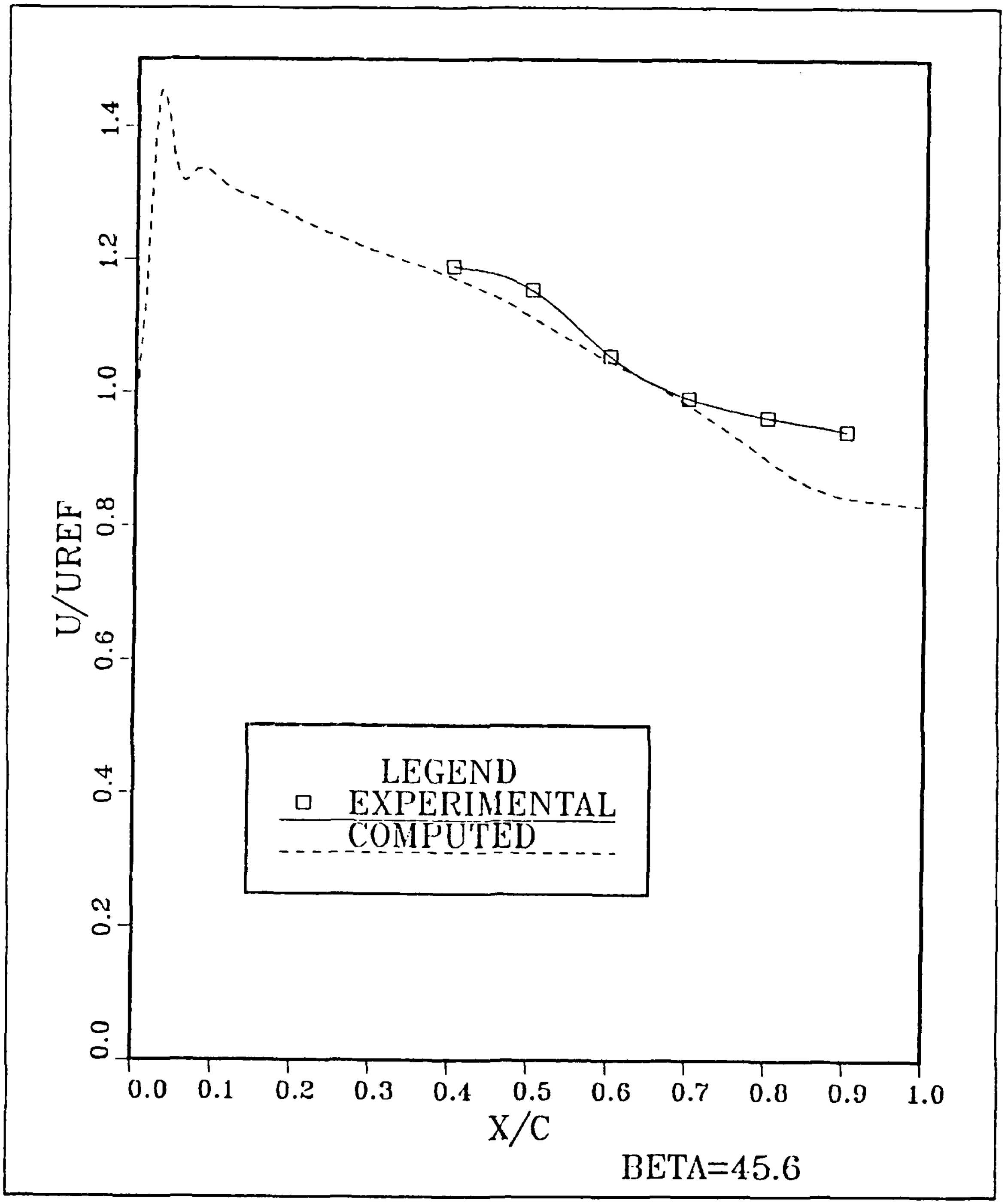

Figure 32. C4 cascade at β = 45.6°: External velocity distribution.

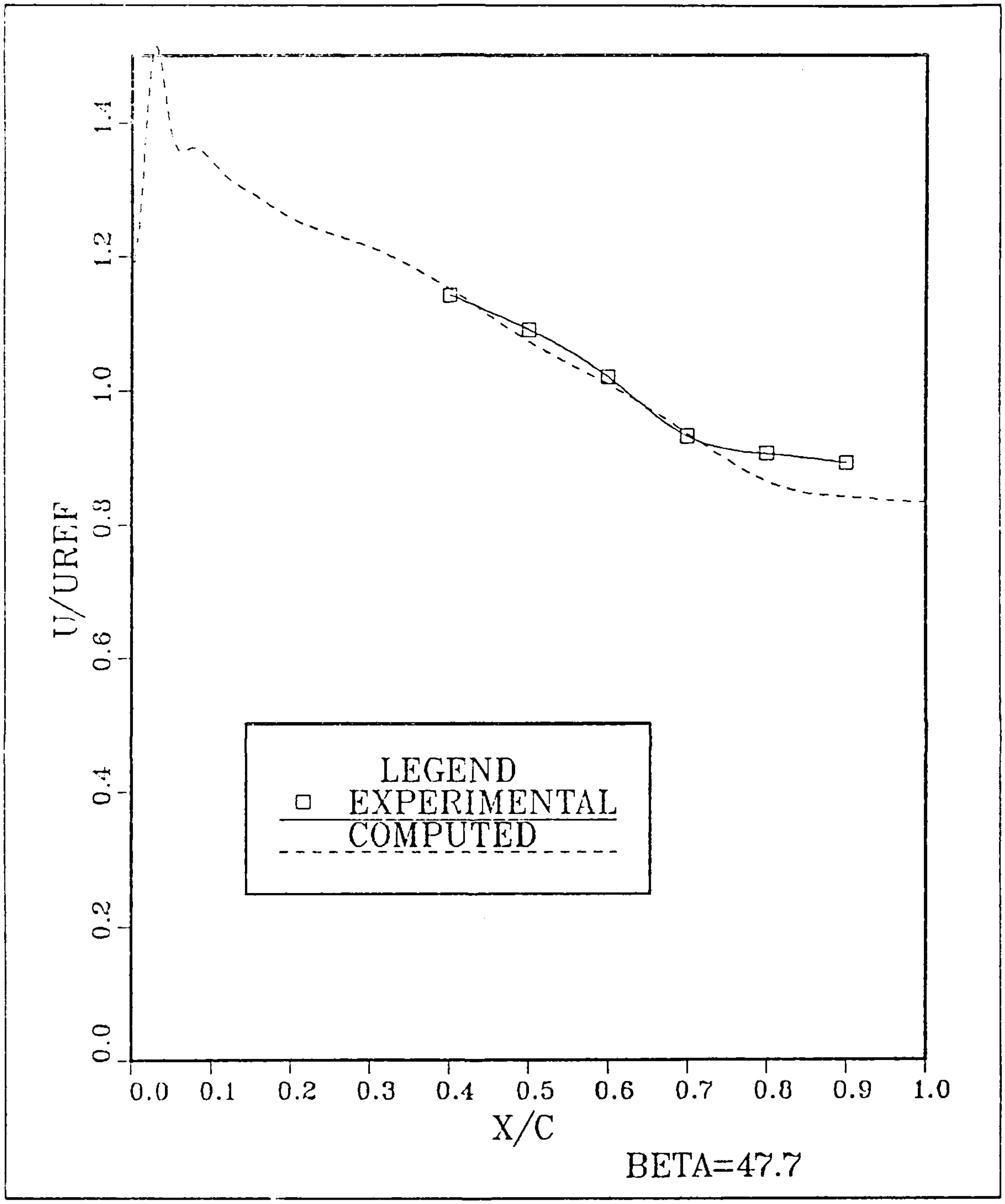

Figure 33. C4 cascade at β = 47.7°: External velocity distribution.

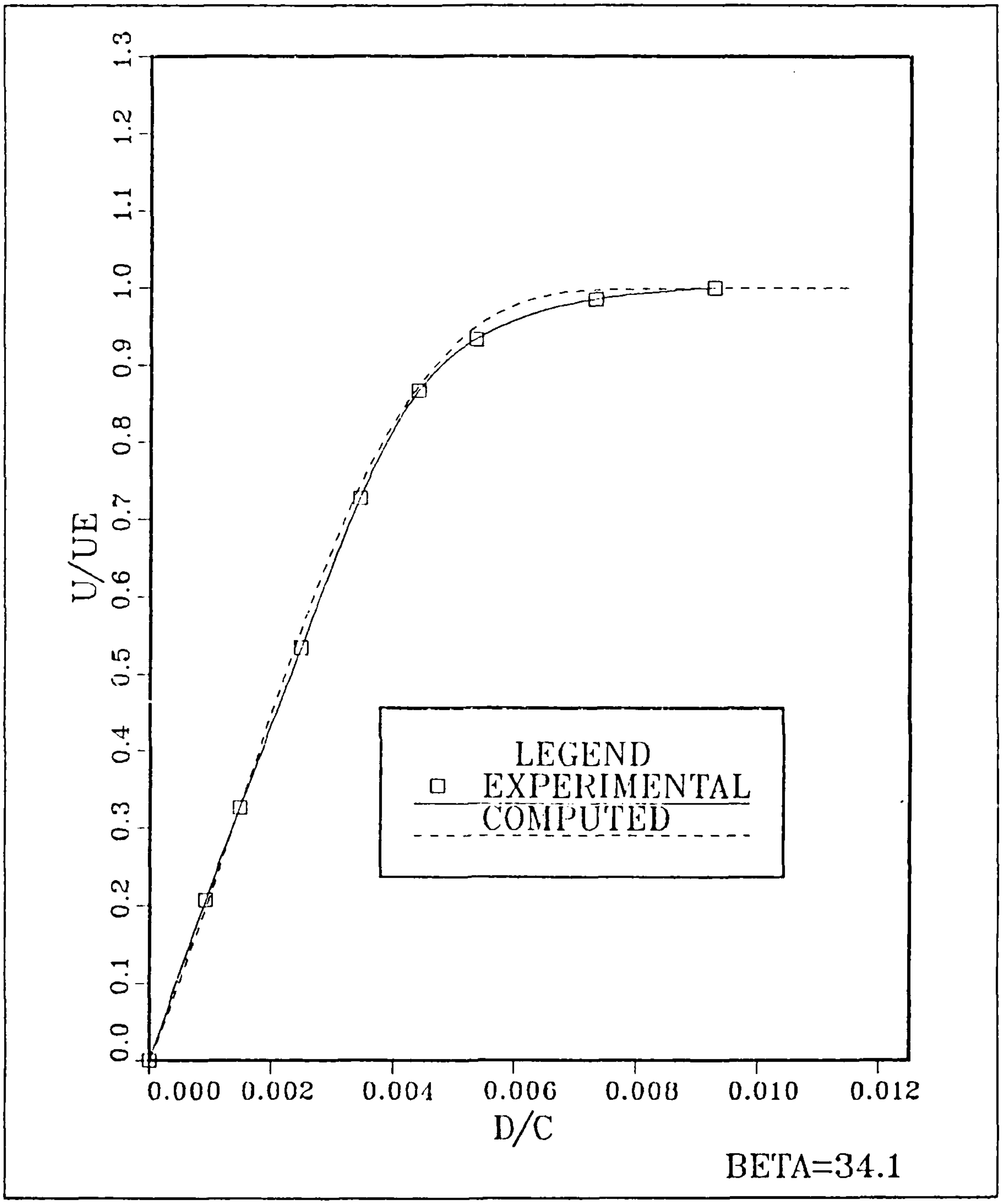

Figure 34. C4 cascade at $\beta = 34.1°$: Velocity profile at 50% chord.

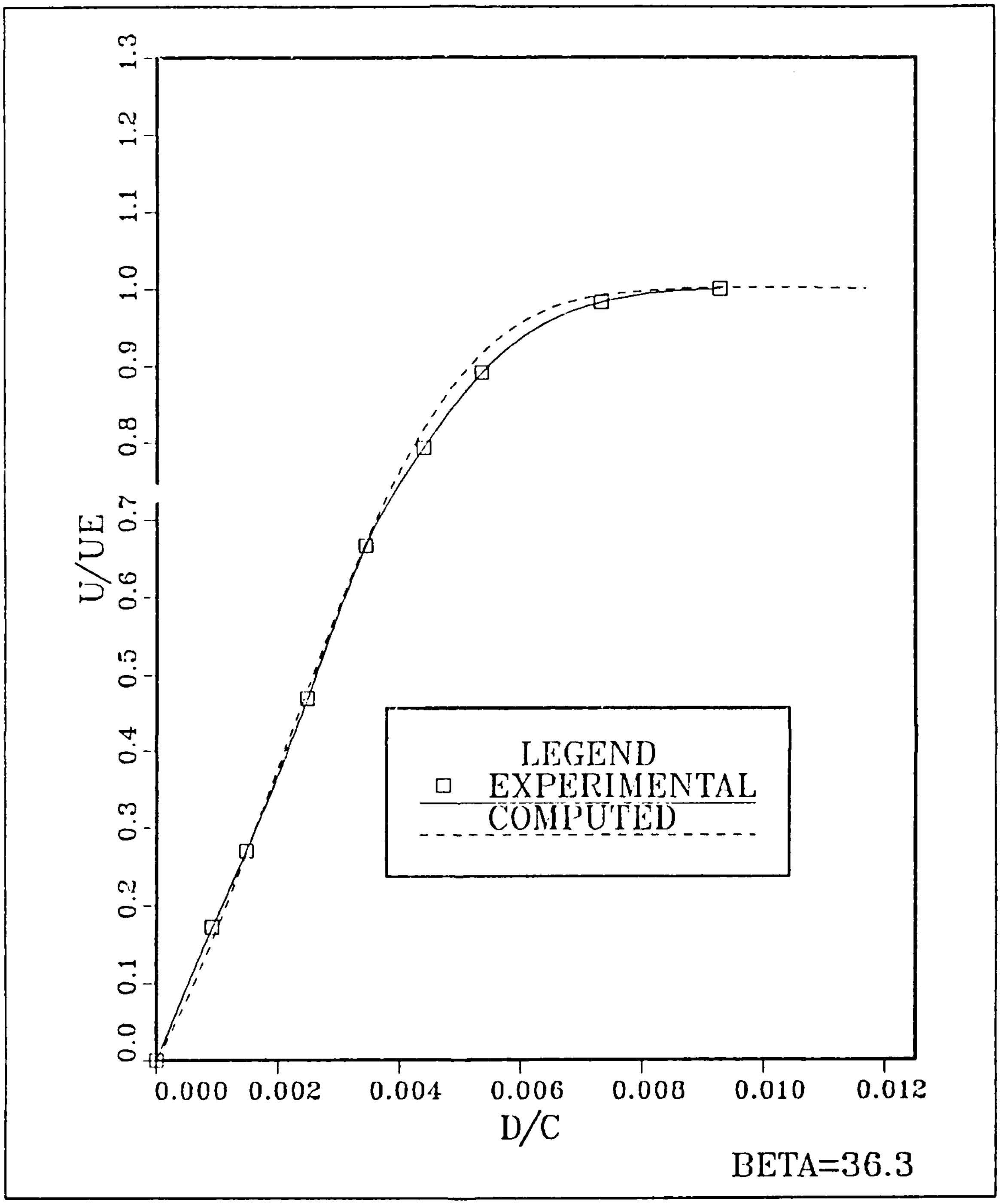

Figure 35. C4 cascade at β = 36.3°: Velocity profile at 50% chord.

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The interactive viscous inviscid computer code, has been investigated by comparing its predictions of boundary layer parameters to experimental data.

It has been found that the code yields reasonable results for lightly loaded cascades, but the prediction of the boundary layer thickness on the suction surface of highly loaded cascades deviates significantly from experimentally measured data. In two cases involving highly cambered cascade blades, with sharp leading edge, the code failed to run. It was also found that the prediction of external velocity distribution on highly loaded cascades was inaccurate.

The main reasons to the discrepancies in the prediction of the boundary layer thickness seem to be:

1. Inaccuracy in predicting flow parameters in regions of large flow separation (due to inadequate transition model and approximations made in calculating the flow in separated areas).
2. Inaccurate turbulence modelling.
3. Possible violation of the basic assumptions of the boundary layer theory in areas of very thick boundary layers.
4. The wake is not calculated by the code. The result is inaccurate flow prediction near the trailing edge.

The inaccuracy in the prediction of the external velocity distribution in highly loaded cascades is due to the interaction law, which does not account for the presence of adjacent blades.

B. RECOMMENDATIONS

The recommended steps in order to improve the code are:

1. Improving the interaction law by assuming a distribution of sources on the actual surface (instead of the assumption of a flat plate), letting the correction term to the external velocity vary across the boundary layer and distributing sources on the adjacent blade as well for better modelling of the boundary layer effect on the external velocity.

2. Changes to the derivation of the boundary layer equations should be investigated to allow a better treatment of thick boundary layers (like omitting the assumption of $\partial p / \partial y = 0$ across the boundary layer).

3. Different turbulence models should be investigated.

4. The wake should be included in the calculations.

APPENDIX A. COMPUTER CODE LISTING

```
C***********************************************************************
C*****    VISCOUS-INVISCID INTERACTION PROGRAM FOR CASCADE FLOWS   *****
C***********************************************************************
C
C                     V E R S I O N    3.A
C                         JANUARY 87
C
C    THIS VISCOUS-INVISCID INTERACTION METHOD, CAPABLE OF COMPUTING BOTH
C    SINGLE AIRFOIL AND CASCADE FLOWS, WAS DEVELOPED BY CEBECI AND
C    COLLABORATEURS AT LONG BEACH STATE AND DOUGLAS AIRCRAFT COMPANY.
C    THE CODE APPLIES TO INCOMPRESSIBLE, 2-DIMENSIONAL, STEADY FLOWS
C    PAST LINEAR, ARBITRARILY STAGGERED CASCADES. THE METHODS BASIC
C    INGREDIENTS INCLUDE
C    1.  A FIRST ORDER PANEL METHOD TO SOLVE LAPLACE'S EQUATION,
C    2.  A FINITE DIFFERENCE SCHEME TO SOLVE THE BOUNDARY LAYER EQUATIONS
C        SUBJECT TO DIRECT OR INTERACTIVE BOUNDARY CONDITIONS,
C    3.  A STRONG INTERACTION MODEL TO COUPLE VISCOUS AND INVISCID FLOW
C        RESULTS, AND
C    4.  A ZERO EQUATION, ALGEBRAIC TURBULENCE MODEL TO ESTIMATE
C        TURBULENT SHEAR STRESSES.
C
C    IN SUMMARY, THE CODE WILL PROVIDE, FOR ATTACHED AS WELL AS MODERATE-
C    LY SEPARATED FLOWS PAST SINGLE AIRFOILS OR CASCADES, THE FOLLOWING
C    1.  INVISCID AND VISCOUS PRESSURE DISTRIBUTIONS,
C    2.  DISTRIBUTIONS OF
C        A.  LOCAL SKIN FRICTION COEFFICIENT,
C        B.  DISPLACEMENT AND MOMENTUM THICKNESS, AND
C    3.  VELOCITY PROFILES ACROSS THE BOUNDARY LAYER.
C
C    MODIFICATIONS SINCE VERSION 3.0:
C    1.  PRECISE ASSIGNMENT OF BEGIN OF TRANSITION.
C    2.  CORRECTION OF AN ERROR IN THE CALCULATION OF MOMENTUM THICKNESS.
C    3.  ADDITIONAL PRINT OPTION: IP=-2 WILL PROVIDE AN INPUT FILE (UNIT
C        NUMBER 12) FOR THE PLOTTING ROUTINE.
C
C
      COMMON /BLCO/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON/BLOW/VN(100)
      COMMON/BLIN/ TITLE(20),XC(100),YC(100),ISG(100),DELS(100),
     +             XCTR,XTR,ISTRP,ICYCLE,ICYTL,XCTRS(2),TRFIND(2)
      COMMON/CASCDE/INLET,SP,SINGLE,ALPHAA,ALPHAI,STAG
      COMMON/TRN/ PGAMTR,OMEGA,RTHETB,RTRANB
      COMMON/PLOT/NVP(2),NXVP(20,2),ICC
      DIMENSION XO(100),YO(100),X(100),Y(100),VCOM(100),DLS(100),
     + XS(100),YS(100),XSTGR(100),YSTGR(100),DBPP(100)
      DIMENSION CASEID(20),XCTRI(2),ITRI(2),NBL(2)
      LOGICAL SINGLE,TRFIND
      TRFIND(1)= .FALSE.
      TRFIND(2)= .FALSE.
```

```
      ICASE  = 0
1     READ(5,5,END=999) TITLE
5     FORMAT(20A4)
      ICASE = ICASE + 1
      REWIND 3
C     READ (5,10)
10    FORMAT(1X)
      READ (5,20) ITRI(1),ITRI(2),IRST,ICYTL,IP
20    FORMAT(16I5)
C      READ (5,10)
      READ (5,25) INLET,ISTAG,ALPHAI,STAG,SP,PGAMTR,OMEGA
25    FORMAT(2I5,5F10.0)
      READ (5,27)RN,XCTRI(1),XCTRI(2),ALPHAA
27    FORMAT(4E10.0)
      IF (IP.EQ.-2) THEN
         READ (5,20) NVP(1),NVP(2)
         IF (NVP(1).NE.0) READ (5,20) (NXVP(I,1),I=1,NVP(1))
         IF (NVP(2).NE.0) READ (5,20) (NXVP(I,2),I=1,NVP(2))
      END IF
      IF (ICASE .EQ. 1) READ (5,20) N,NI
      IREAD = 1
      IBLOW = 1
      SINGLE = .FALSE.
      IF (SP .LE. 0.0) SINGLE = .TRUE.
      N = N - 1
      N1= N + 1
      IF (ICASE .GT. 1) THEN
      N1 = N1SAVE
      N = N - 1
      GOTO 53
      END IF
      IF (IREAD .EQ. 1) GO TO 40
C      READ (5,10)
      READ (5,30) (XO(I), YO(I), I=1,N+1)
30    FORMAT(2F10.0)
      GO TO 50
C
C40    READ(5,10)
40    READ(5,45) (XO(I) , I=1,N+1)
C      READ(5,10)
      READ(5,45) (YO(I) , I=1,N+1)
45    FORMAT(6F10.0)
C
50    CONTINUE
      IF (IP.EQ.-2) THEN
         WRITE(12,20) N+1,NVP(1),NVP(2),90,70,INLET
         IF (NVP(1).NE.0) WRITE(12,20) (NXVP(I,1),I=1,NVP(1))
         IF (NVP(2).NE.0) WRITE(12,20) (NXVP(I,2),I=1,NVP(2))
         IF (INLET.NE.1) WRITE(12,80) RN,ALPHAA
         IF (INLET.EQ.1) WRITE(12,80) RN,ALPHAI
         WRITE(12,82) (XO(I),I=1,N+1)
         WRITE(12,82) (YO(I),I=1,N+1)
   80    FORMAT(2E15.5)
   82    FORMAT(8F10.6)
      END IF
C
```

```
      NRITE = (NI+1)/2
      IMIN = (N1-1)/2+1
      IF((N1/2*2) .EQ. N1) IMIN = N1/2
      CALL TRGRID (N1,XO,YO,NI,NRITE,0.5,IMIN,RAD,1,NXSS1)
      N1SAVE = N1
53    CONTINUE
      ALPHAA = 0.0174533 * ALPHAA
      ALPHAI = 0.0174533 * ALPHAI
      STAG   = 0.0174533 * STAG
C
      IF (INLET .EQ. 0) THEN
      ALPHA = ALPHAA
      ELSE
      ALPHA = ALPHAI
      END IF
C
      IF (ISTAG .NE. 0) THEN
      CALL STAGR(N1,STAG,XO,YO,XSTGR,YSTGR)
      ELSE
      DO 55 I = 1 , N1
      XSTGR(I) = XO(I)
      YSTGR(I) = YO(I)
55    CONTINUE
      END IF
C
C  READ DATA FROM VISCOUS CAL.
C
      ICYCLE = 0
60    ICYCLE = ICYCLE + 1
C
      CALL POTNL(N1,IRST,ALPHA,CHORD,XO,YO,XSTGR,YSTGR,X,Y,DLS,VCOM,
     +           DBPP)
      IF (ICYCLE .GT. ICYTL) THEN
      REWIND 3
      WRITE (3)N1,(XO(I),YO(I),DLS(I),VN(I),DBPP(I),I=1,N1)
      GOTO 1
      END IF
C
      IF (ISTAG .NE. 0) THEN
      DO 70 I = 1 , N1-1
      X(I) = 0.5 * (XO(I)+XO(I+1))
      Y(I) = 0.5 * (YO(I)+YO(I+1))
70    CONTINUE
      END IF
C
      CALL CASBLP(N1,XO,YO,X,Y,XS,YS,DLS,VCOM,DBPP,RN
     +            ,NBL,ITRI,XCTRI,TITLE)
      GO TO 60
999   CONTINUE
      STOP
      END
C
      SUBROUTINE POTNL(N1,IRST,ALPHA,CHORD,XO,YO,XSTGR,YSTGR,X,Y,DLS,
     +                 VCOM,DBPP)
C
      COMMON/BLOW/VN(100)
```

```
      COMMON /BLCO/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP                   INT01620
      COMMON/BLIN/TITLE(20),XC(100),YC(100),ISG(100),DELS(100),        INT01630
     +             XCTR,XTR,ISTRP,ICYCLE,ICYTL,XCTRS(2),TRFIND(2)      INT01640
      COMMON/CASCDE/ INLET,SP,SINGLE,ALPHAA,ALPHAI,STAG                INT01650
C   SIMPLE SOURCE POTENTIAL CODE                                       INT01660
      DIMENSION AOFF(100,100), BOFF(100,100), XP(100), YP(100),X(100), INT01670
     +     S(100),C(100), D(100),VTAN(3,100),VNOR(3,100),R(3,100),     INT01680
     +     VCOM(100),SIGCOM(100),CP(100),XO(100),YO(100)               INT01690
     +    ,VNC(100),D1(100),D2(100),D3(100),SO(100),SC(100)            INT01700
     +    ,XOFF(100),YOFF(100),T(3,100),VTCOM(100),VNCOM(100)          INT01710
     +    ,XS(100),YS(100),SOFF(100),COFF(100),XPOFF(100),YPOFF(100)   INT01720
     +    ,Y(100),SIG(3,100),DLS(100),DLSC(100),A(100,100),B(100,100)  INT01730
     +    ,XSTGR(100),YSTGR(100),VUT(3),VLT(3),VUN(3),VLN(3),DBPP(100) INT01740
     +    ,CPI(100),XOS(100),YOS(100),DBPPC(100)                       INT01750
      REAL NUM1 , NUM2                                                 INT01760
      LOGICAL OFF,SINGLE                                               INT01770
      OFF = .FALSE.                                                    INT01780
      PI = 3.141592                                                    INT01790
      CM = 0.0                                                         INT01800
      N = N1 - 1                                                       INT01810
      IF (ICYCLE .EQ. 1) THEN                                          INT01820
      IF (IRST .EQ. 0) THEN                                            INT01830
      DO 10 I=1,N1                                                     INT01840
      DLS(I) = 0.0                                                     INT01850
      VN (I) = 0.0                                                     INT01860
      DBPP(I)= 0.0                                                     INT01870
10    CONTINUE                                                         INT01880
      ELSE                                                             INT01890
      DO 5 I = 1 , N1                                                  INT01900
      XOS(I) = XO(I)                                                   INT01910
      YOS(I) = YO(I)                                                   INT01920
5     CONTINUE                                                         INT01930
      READ (3) NT,(XS(I),YS(I),DLSC(I),VNC(I),DBPPC(I),I=1,NT)         INT01940
      XMIN = XS(1)                                                     INT01950
      DO 15 I = 2 , NT                                                 INT01960
      IF (XS(I) .GT. XMIN) GOTO 15                                     INT01970
      XMIN = XS(I)                                                     INT01980
      IMIN = I                                                         INT01990
15    CONTINUE                                                         INT02000
      DO 17 I = 1 , NT                                                 INT02010
      IF (I .LT. IMIN) GOTO 16                                         INT02020
      XS(I) = XS(I) - XMIN                                             INT02030
      GOTO 17                                                          INT02040
16    XS(I) = XMIN - XS(I)                                             INT02050
17    CONTINUE                                                         INT02060
C                                                                      INT02070
      XMIN = XOS(I)                                                    INT02080
      DO 20 I = 2 , N1                                                 INT02090
      IF (XOS(I) .GT. XMIN) GOTO 20                                    INT02100
      XMIN = XOS(I)                                                    INT02110
      IMIN = I                                                         INT02120
20    CONTINUE                                                         INT02130
      DO 22 I = 1 , N1                                                 INT02140
      IF (I .LT. IMIN) GOTO 21                                         INT02150
      XOS(I) = XOS(I) - XMIN                                           INT02160
```

```
      GOTO 22
21    XOS(I) = XMIN - XOS(I)
22    CONTINUE
C
      CALL DIFF3(NT,XS,DLSC,D1,D2,D3,0)
      CALL INTRP3(NT,XS,DLSC,D1,D2,D3,N1,XOS,DLS)
      CALL AMEAN (1,N1,XOS,DLS,1)
      CALL DIFF3 (NT,XS,VNC,D1,D2,D3,0)
      CALL INTRP3(NT,XS,VNC,D1,D2,D3,N1,XOS,VN)
      CALL AMEAN (1,N1,XOS,VN,1)
      CALL DIFF3 (NT,XS,DBPPC,D1,D2,D3,0)
      CALL INTRP3(NT,XS,DBPPC,D1,D2,D3,N1,XOS,DBPP)
      CALL AMEAN (1,N1,XOS,DBPP,1)
      END IF
      END IF
      DO 30 I=1,N1
      XP(I) = XSTGR(I)
      YP(I) = YSTGR(I)
30    CONTINUE
C  CALCULATE GEOMETRIC QUANTITIES
      DO 100 J=1,N
      VNC(J) = 0.5 * (VN(J) + VN(J+1))
      X(J)= .5*(XP(J)+XP(J+1))
      Y(J)= .5*(YP(J)+YP(J+1))
      D(J)= SQRT((XP(J+1)-XP(J))**2 + (YP(J+1)-YP(J))**2)
      C(J)= (XP(J+1)-XP(J))/D(J)
      S(J)= (YP(J+1)-YP(J))/D(J)
100   CONTINUE
C
      IF ( INLET .NE. 0 .AND. .NOT. SINGLE) THEN
      SUM = D(1)
      DO 35 J = 2, N
      SUM = SUM + D(J)
35    CONTINUE
      Q = 2.0 * PI * SUM / SP
      ELSE
      Q = 0.0
      END IF
C
C  CALCULATE NORMAL AND TANGENTIAL MATRICES
102   CONTINUE
      IF (SINGLE) THEN
      IF ( .NOT. OFF) THEN
      DO 120 I=1,N
      DO 110 J=1,N
      IF (J .EQ. I) GO TO 105
      XX= (X(I)-X(J))*C(J) + (Y(I)-Y(J))*S(J)
      YY=-(X(I)-X(J))*S(J) + (Y(I)-Y(J))*C(J)
      UU= LOG(((XX+.5*D(J))**2+YY**2)/((XX-.5*D(J))**2+YY**2))
      VV= 2.*ATAN2(YY*D(J), XX**2+YY**2-(.5*D(J))**2)
      SS= S(I)*C(J) - C(I)*S(J)
      CC= C(I)*C(J) + S(I)*S(J)
      A(I,J)= -UU*SS + VV*CC
      B(I,J)=  UU*CC + VV*SS
      GO TO 110
105   A(I,J) = 6.2831853
```

```
      B(I,J) = 0.0                                                      INT02730
  110 CONTINUE                                                          INT02740
  120 CONTINUE                                                          INT02750
      ELSE                                                              INT02760
      DO 140 I=1,N                                                      INT02770
      DO 130 J=1,N                                                      INT02780
      XX= (XOFF(I)-X(J))*C(J) + (YOFF(I)-Y(J))*S(J)                     INT02790
      YY=-(XOFF(I)-X(J))*S(J) + (YOFF(I)-Y(J))*C(J)                     INT02800
      UU= LOG(((XX+.5*D(J))**2+YY**2)/((XX-.5*D(J))**2+YY**2))          INT02810
      VV= 2.*ATAN2(YY*D(J), XX**2+YY**2-(.5*D(J))**2)                   INT02820
      SS= SOFF(I)*C(J) - COFF(I)*S(J)                                   INT02830
      CC= COFF(I)*C(J) + SOFF(I)*S(J)                                   INT02840
      AOFF(I,J)= -UU*SS + VV*CC                                         INT02850
      BOFF(I,J)=  UU*CC + VV*SS                                         INT02860
  130 CONTINUE                                                          INT02870
  140 CONTINUE                                                          INT02880
      END IF                                                            INT02890
      ELSE                                                              INT02900
      IF ( .NOT. OFF) THEN                                              INT02910
      DO 50 I=1,N                                                       INT02920
      DO 40 J=1,N                                                       INT02930
      IF (J .EQ. I) GO TO 45                                            INT02940
      XX= (X(I)-X(J))*C(J) + (Y(I)-Y(J))*S(J)                           INT02950
      YY=-(X(I)-X(J))*S(J) + (Y(I)-Y(J))*C(J)                           INT02960
      DX1 = PI *(X(I)-XP(J)) / SP                                       INT02970
      DY1 = PI *(Y(I)-YP(J)) / SP                                       INT02980
      DX2 = PI *(X(I)-XP(J+1)) / SP                                     INT02990
      DY2 = PI *(Y(I)-YP(J+1)) / SP                                     INT03000
      R1SQ = (COSH(DX1))**2 - (COS(DY1))**2                             INT03010
      R2SQ = (COSH(DX2))**2 - (COS(DY2))**2                             INT03020
      UU   = LOG(R1SQ/R2SQ)                                             INT03030
      NUM1 = DX1 * COSH(DX1) * SIN(DY1) -                               INT03040
     +       DY1 * SINH(DX1) * COS(DY1)                                 INT03050
      DNUM1= DX1 * SINH(DX1) * COS(DY1) +                               INT03060
     +       DY1 * COSH(DX1) * SIN(DY1)                                 INT03070
      NUM2 = DX2 * COSH(DX2) * SIN(DY2) -                               INT03080
     +       DY2 * SINH(DX2) * COS(DY2)                                 INT03090
      DNUM2= DX2 * SINH(DX2) * COS(DY2) +                               INT03100
     +       DY2 * COSH(DX2) * SIN(DY2)                                 INT03110
      EXV  = 2.0 * ATAN2(NUM2,DNUM2) - 2.0 * ATAN2(NUM1,DNUM1)          INT03120
      VV= 2.*ATAN2(YY*D(J), XX**2+YY**2-(.5*D(J))**2)                   INT03130
      VV = VV + EXV                                                     INT03140
      SS= S(I)*C(J) - C(I)*S(J)                                         INT03150
      CC= C(I)*C(J) + S(I)*S(J)                                         INT03160
      A(I,J)= -UU*SS + VV*CC                                            INT03170
      B(I,J)=  UU*CC + VV*SS                                            INT03180
      GO TO 40                                                          INT03190
   45 A(I,J) = 6.2831853                                                INT03200
      B(I,J) = 0.0                                                      INT03210
   40 CONTINUE                                                          INT03220
   50 CONTINUE                                                          INT03230
      ELSE                                                              INT03240
      DO 70 I=1,N                                                       INT03250
      DO 60 J=1,N                                                       INT03260
      XX= (XOFF(I)-X(J))*C(J) + (YOFF(I)-Y(J))*S(J)                     INT03270
      YY=-(XOFF(I)-X(J))*S(J) + (YOFF(I)-Y(J))*C(J)                     INT03280
```

```
      DX1 = PI *(XOFF(I)-XP(J)) / SP
      DY1 = PI *(YOFF(I)-YP(J)) / SP
      DX2 = PI *(XOFF(I)-XP(J+1)) / SP
      DY2 = PI *(YOFF(I)-YP(J+1)) / SP
      R1SQ = (COSH(DX1))**2 - (COS(DY1))**2
      R2SQ = (COSH(DX2))**2 - (COS(DY2))**2
      UU   = LOG(R1SQ/R2SQ)
      NUM1 = DX1 * COSH(DX1) * SIN(DY1) -
     +       DY1 * SINH(DX1) * COS(DY1)
      DNUM1= DX1 * SINH(DX1) * COS(DY1) +
     +       DY1 * COSH(DX1) * SIN(DY1)
      NUM2 = DX2 * COSH(DX2) * SIN(DY2) -
     +       DY2 * SINH(DX2) * COS(DY2)
      DNUM2= DX2 * SINH(DX2) * COS(DY2) +
     +       DY2 * COSH(DX2) * SIN(DY2)
      EXV  = 2.0 * ATAN2(NUM2,DNUM2) - 2.0 * ATAN2(NUM1,DNUM1)
      VV= 2.*ATAN2(YY*D(J), XX**2+YY**2-(.5*D(J))**2)
      VV = VV + EXV
      SS= SOFF(I)*C(J) - COFF(I)*S(J)
      CC= COFF(I)*C(J) + SOFF(I)*S(J)
      AOFF(I,J)= -UU*SS + VV*CC
      BOFF(I,J)=  UU*CC + VV*SS
60    CONTINUE
70    CONTINUE
      END IF
      END IF
C  NORMAL AND TANGENTIAL COMPONENTS OF FUNDAMENTAL SOLUTIONS
C
      DO 160 I=1,N
      SUMR= 0.
      SUMT= 0.
      IF (  .NOT. OFF) THEN
      R(1,I)= S(I)+VNC(I)/COS(ALPHA)
      T(1,I)= C(I)
      R(2,I)= -C(I)
      T(2,I)= S(I)
      DO 145 J=1,N
      SUMR = SUMR + B(I,J)
      SUMT = SUMT + A(I,J)
145   CONTINUE
      ELSE
      R(1,I) = SOFF(I)
      T(1,I) = COFF(I)
      R(2,I) =-COFF(I)
      T(2,I) = SOFF(I)
      DO 150 J=1,N
      SUMR= SUMR + BOFF(I,J)
      SUMT= SUMT + AOFF(I,J)
150   CONTINUE
      END IF
      R(3,I)= SUMR
      T(3,I)= SUMT
160   CONTINUE
C
      IF ( OFF ) GO TO 275
C     DECOMPOSITION OF MATRIX A
```

```
      DO 230 I=1,N-1
      DO 220 K=I+1,N
      A(K,I)= A(K,I)/A(I,I)
      DO 210 J=I+1,N
      A(K,J)= A(K,J)- A(K,I)*A(I,J)
210   CONTINUE
220   CONTINUE
230   CONTINUE
C  OPERATE ON FUNDAMENTAL RIGHT SIDES WITH LOWER TRIANGULAR
      DO 270 K=1,3
      DO 260 J=1,N-1
      DO 250 I=J+1,N
      R(K,I)= R(K,I) - A(I,J)*R(K,J)
250   CONTINUE
260   CONTINUE
270   CONTINUE
C  BACK SOLUTION
      DO 300 K=1,3
      DO 290 I=N,1,-1
      SUM= 0.
      DO 280 J=N,I+1,-1
      SUM= SUM + A(I,J)*SIG(K,J)
280   CONTINUE
      SIG(K,I)= (R(K,I)-SUM)/A(I,I)
290   CONTINUE
300   CONTINUE
      OFF = .TRUE.
C
C    ADD DIS-PLACE VERTICALLY TO THE BODY TO GENERATE
C    DISPLACEMENT SURFACE
C
      DO 305 I = 2 , N
      SOFF(I) = (S(I)*D(I-1)+S(I-1)*D(I))/(D(I-1)+D(I))
      COFF(I) = (C(I)*D(I-1)+C(I-1)*D(I))/(D(I-1)+D(I))
305   CONTINUE
      SOFF(1) = 2.0*S(1) - SOFF(2)
      SOFF(N1)= 2.0*S(N) - SOFF(N)
      COFF(1) = 2.0*C(1) - COFF(2)
      COFF(N1)= 2.0*C(N) - COFF(N)
      DO 306 I = 1 , N1
      XPOFF(I) = XP(I) - SOFF(I) * DLS(I)
      YPOFF(I) = YP(I) + COFF(I) * DLS(I)
306   CONTINUE
      DO 307 I = 1 , N
      XOFF(I) = 0.5 * (XPOFF(I) + XPOFF(I+1))
      YOFF(I) = 0.5 * (YPOFF(I) + YPOFF(I+1))
      DOFF = SQRT((XPOFF(I+1)-XPOFF(I))**2 +
     +            (YPOFF(I+1)-YPOFF(I))**2 )
      COFF(I) = (XPOFF(I+1)-XPOFF(I))/DOFF
      SOFF(I) = (YPOFF(I+1)-YPOFF(I))/DOFF
307   CONTINUE
      GO TO 102
C  CALCULATION OF SURFACE VELOCITIES FOR THE FUNDAMENTAL SOLUTIONS
C
275   DO 330 K=1,3
      DO 320 I=1,N
```

```
      SUMT= T(K,I)
      SUMN=-R(K,I)
      DO 310 J=1,N
      SUMT= SUMT + BOFF(I,J)*SIG(K,J)
      SUMN= SUMN + AOFF(I,J)*SIG(K,J)
310   CONTINUE
      VTAN(K,I)= SUMT
      VNOR(K,I)= SUMN
320   CONTINUE
C
      DOFF1 = SQRT((XPOFF(2)-XPOFF(1))**2+(YPOFF(2)-YPOFF(1))**2)
      DOFF2 = SQRT((XPOFF(3)-XPOFF(2))**2+(YPOFF(3)-YPOFF(2))**2)
      DOFFN = SQRT((XPOFF(N+1)-XPOFF(N))**2+(YPOFF(N+1)-YPOFF(N))
     +             **2)
      DOFFN1= SQRT((XPOFF(N)-XPOFF(N-1))**2+(YPOFF(N)-YPOFF(N-1))
     +             **2)
      VUT(K) = VTAN(K,N) + DOFFN * (VTAN(K,N)-VTAN(K,N-1))/
     +                             (DOFFN+DOFFN1)
      VUN(K) = VNOR(K,N) + DOFFN * (VNOR(K,N)-VNOR(K,N-1))/
     +                             (DOFFN+DOFFN1)
      VLT(K) = VTAN(K,1) + DOFF1 * (VTAN(K,1)-VTAN(K,2))/
     +                             (DOFF1+DOFF2)
      VLN(K) = VNOR(K,1) + DOFF1 * (VNOR(K,1)-VNOR(K,2))/
     +                             (DOFF1+DOFF2)
330   CONTINUE
C  OUTPUT FUNDAMENTAL SOLUTIONS
      IF (ICYCLE .EQ. 1 .OR. ICYCLE .GE. ICYTL-1 .OR. IP .GE. 0)
     + WRITE(6,335) TITLE
335   FORMAT(1H1,///20A4//)
C     DO 360 K=1,3
C     WRITE (6,340) K
C340   FORMAT(////,1H ,'FUNDAMENTAL SOLUTION NUMBER ',I2////)
C      WRITE(6,345)
345   FORMAT(3X,'I',8X,'X',11X,'Y',10X,'VT',10X,'VN',8X,'SIG' ///)
C      WRITE(6,375) 1,  XP(1) , YP(1), VLT(K),VLN(K)
C      DO 350 I=1,N
C      WRITE(6,375) I , X(I), Y(I), VTAN(K,I),VNOR(K,I),SIG(K,I)
C350   CONTINUE
C      WRITE(6,375) N1 , XP(N1),YP(N1),VUT(K),VUN(K)
C360   CONTINUE
C  COMBINED FLOW AT ANGLE OF ATTACK
C
      IF ( INLET .NE. 0) THEN
      YYY = ((VUT(3)+VLT(3))*TAN(ALPHAI)+(VUT(1)+VLT(1))*Q)/
     +      ((VUT(3)+VLT(3))-(VUT(2)+VLT(2))*Q)
      XXX = -((VUT(1)+VLT(1))+(VUT(2)+VLT(2))*TAN(ALPHAI))/
     +       ((VUT(3)+VLT(3))-(VUT(2)+VLT(2))*Q)
      ALPHA = ACOS(1.0/SQRT(1.0+YYY**2))
      COSAL = COS(ALPHA)
      SINAL = SIN(ALPHA)
      W = XXX/SQRT(1.0+YYY**2)
      ELSE
      COSAL = COS(ALPHA)
      SINAL = SIN(ALPHA)
      W=-((VLT(1)+VUT(1))*COSAL+(VLT(2)+VUT(2))*SINAL)/
     +       (VLT(3)+VUT(3))
```

```
      END IF
C  FORCE COEFFICIENT CALCULATION
      SUM1= 0.
      SUMX= 0.
      SUMY= 0.
      DO 390 I=1,N
      SUM1= SUM1+ D(I)
      SUMX= SUMX- VCOM(I)**2*S(I)*D(I)
      SUMY= SUMY+ VCOM(I)**2*C(I)*D(I)
390   CONTINUE
C  FIND MAN. CHORD LENGTH
      XOMIN = XO(1)
      DO 395 I = 2 , N1
      IF ( XO(I) .GT. XOMIN) GOTO 395
      XOMIN = XO(I)
395   CONTINUE
      CHORD = XO(N1) - XOMIN
      CL1= SUM1*25.13274*W/CHORD
      CL2= (SUMY*COSAL-SUMX*SINAL)/CHORD
      CD = (SUMX*COSAL+SUMY*SINAL)/CHORD
C
C  CALCULATING PARAMETERS FOR INLET VELOCITY AS MODULUS OF NOMORIZED VEL
C
      IF (.NOT. SINGLE) THEN
      NUM1 = SIN(ALPHA)+CL1*CHORD/(4.0*SP)
      ALPHID = ATAN2(NUM1,COS(ALPHA))
      NUM1 = SIN(ALPHA)-CL1*CHORD/(4.0*SP)
      ALPHED = ATAN2(NUM1,COS(ALPHA))
      NUM1 = CL1*CHORD/(2.0*SP)*COS(ALPHA)
      DNUM1= 1.0-(CL1*CHORD/(4.0*SP))**2
      DALPHA = ATAN2(NUM1,DNUM1)
      UOUI = (TAN(ALPHID)-TAN(ALPHED))*(2.0*SP/CHORD*COS(ALPHID))/CL1
      CLI = CL1 * UOUI**2
      UIOU = 1.0/UOUI
      VEXIT = COS(ALPHA)/COS(ALPHED)
      ELSE
      ALPHID = ALPHA
      ALPHED = ALPHA
      DALPHA = 0.0
      UOUI = 1.0
      UIOU = 1.0
      CLI  = CL1
      VEXIT = 1.0
      END IF
      FAC = 180.0/PI
      ALPHID = ALPHID * FAC
      ALPHED = ALPHED * FAC
      DALPHA = DALPHA * FAC
      ALPHAD = ALPHA  * FAC
      IF (ICYCLE .EQ. 1 .OR. ICYCLE .GE. ICYTL-1 .OR. IP .GE. 0) THEN
      WRITE(6,370) ALPHAD ,ALPHID,ALPHED,DALPHA,UIOU,VEXIT
370   FORMAT (////,1H , 'COMBINED FLOW AT AVERAGE ANGLE OF ATTACK = ',
     +          F8.3, 4X,'DEGREES', /,1H ,17X,'INLET ANGLE OF ',
     +         'ATTACK   = ',F8.3,4X,'DEGREES',/,1H ,
     +          17X,'EXIT ANGLE =  ',F8.3,4X,'DEGREES',/,1H ,17X,
     +          'TURNNING ANGLE = ',F8.3,4X,'DEGREES',/,1H ,17X,
```

```
     +           'INLET VEL = ',F10.6,3X,'EXIT VEL = ',F10.6,//)
      WRITE(6,365)
365   FORMAT(3X,'I',8X,'XO',10X,'YO',10X,'X',11X,'Y',10X,'VT',
     +             10X,'VN',11X,'V',10X,'CP',9X,'CPI'///)
      END IF
      DO 380 I=1,N
      VTCOM(I)= VTAN(1,I)*COSAL+VTAN(2,I)*SINAL+W*VTAN(3,I)
      VNCOM(I)= VNOR(1,I)*COSAL+VNOR(2,I)*SINAL+W*VNOR(3,I)
      VCOM(I)= SQRT(VTCOM(I)**2 + VNCOM(I)**2)
      IF (VTCOM(I) .LT. 0.0) VCOM(I) = -VCOM(I)
      CP(I) = 1.0 - VCOM(I) ** 2
      CPI(I)= 1.0 - (VCOM(I)*UOUI)**2
      SIGCOM(I) = SIG(1,I)*COSAL+SIG(2,I)*SINAL+W*SIG(3,I)
      XP(I) = 0.5 * (XO(I)+XO(I+1))
      YP(I) = 0.5 * (YO(I)+YO(I+1))
380   CONTINUE
      IF (ICYCLE .EQ. 1 .OR. ICYCLE .GE. ICYTL-1 .OR. IP .GE. 0) THEN
      WRITE (1,374)  ( XO(I),YO(I),XP(I),YP(I),CP(I),CPI(I) ,I=1,N)
      WRITE (6,375)  ( I, XO(I), YO(I), XP(I), YP(I), VTCOM(I),
     +                 VNCOM(I),VCOM(I),CP(I), CPI(I) ,I=1,N)
374   FORMAT(6F10.4)
375   FORMAT(1X, I3, 9F12.4)
C     WRITE (2) I,XO(I),YO(I),XSTGR(I),YSTGR(I),DLS(I),X(I),Y(I),VCOM(I)
      WRITE(6,385) N+1,XO(N+1),YO(N+1)
385   FORMAT(1X,I3,2F12.4)
      WRITE(6,400) CHORD, CL1 ,CLI
400   FORMAT(///3X,'CHORD = ',F10.5,4X,'CL(AVG) = ',F10.5,4X,
     +           'CL(INLET) = ',F10.5)
      END IF
420   FORMAT(/3X,1HI,6X,2HSO,10X,2HSC,9X,3H VN,9X,3HVNC,9X,3HDLS,8X,
     +            4HDLSC)
430   FORMAT(I5,6E12.4)
      RETURN
      END
C
C  ----------------------------------------
C           DATA SET KCBCAMEAN  AT LEVEL 001 AS OF 08/24/84
C           DATA SET KCBCAMEAN  AT LEVEL 003 AS OF 04/05/84
      SUBROUTINE AMEAN(NS,ND,X,Y,IT)
C
C    SMOOTH DATA USING 3-PTS WEIGHTING FORMULA
C    NS    :  STARTING PINT OF THE DATA TO BE SMOOTHED
C    ND    :  END PINT OF THE DATA TO BE SMOOTHED
C    X, Y :  INDEPENDENT + DEPENDENT VARAIBLES OF THE DATA
C             TO BE SMOOTHED
C    IT    :  CYCLES OF DATA SMOOTHING
C
      DIMENSION X(101),Y(101)
C
C -------------------------------------------------------
C
      NM    = ND -NS
      IF(NM .LT.2 .OR. IT .LT.1) RETURN
C
      NDM1  = ND - 1
      NSP1  = NS + 1
```

```
      DO 20 K=1,IT                                                        INT06090
      DL1    = X(NSP1) - X(NS)                                            INT06100
      Y1     = Y(NS)                                                      INT06110
      DO 10 I=NSP1,NDM1                                                   INT06120
      DL2    = X(I + 1) -X(I)                                             INT06130
      Y2     = Y(I)                                                       INT06140
      YM     = (DL2 * Y1 + DL1 * Y(I+1))/(DL1 + DL2)                      INT06150
      Y(I)   = 0.5 * (Y2 + YM)                                            INT06160
      DL1    = DL2                                                        INT06170
      Y1     = Y2                                                         INT06180
   10 CONTINUE                                                            INT06190
   20 CONTINUE                                                            INT06200
C                                                                         INT06210
      RETURN                                                              INT06220
      END                                                                 INT06230
C          DATA SET KCBCBLGRID AT LEVEL 001 AS OF 08/24/84                INT06240
C          DATA SET KCBCBLGRID AT LEVEL 001 AS OF 08/24/84                INT06250
C          DATA SET KCBCBLGRID AT LEVEL 004 AS OF 04/05/84                INT06260
      SUBROUTINE BLGRID(N,X,T,D1)                                         INT06270
C                                                                         INT06280
C    GENERATE B. L. X-WISE GRID USING MODIFIED COSINE DISTRIBUTION        INT06290
C                                                                         INT06300
      DIMENSION X(101),T(101),D1(101)                                     INT06310
      DATA CRAD/57.2957795/, BPI/3.14159265/                              INT06320
C                                                                         INT06330
C    ------------------------------------------------------------         INT06340
C                                                                         INT06350
      NN      = 2 * N - 1                                                 INT06360
      EN      = FLOAT((NN-1)/2)                                           INT06370
      THO     = 10./CRAD                                                  INT06380
      CTO1    = 1. + COS(THO)                                             INT06390
      DTH     = (BPI - THO) / EN                                          INT06400
      FI      = FLOAT(N - 2)                                              INT06410
      DO 10 I=N,NN                                                        INT06420
      FI      = 1.0 + FI                                                  INT06430
      II      = I - N + 1                                                 INT06440
      XII     = THO + FI * DTH                                            INT06450
      X(II)   = (1.0 + COS(XII))/CTO1                                     INT06460
   10 CONTINUE                                                            INT06470
      X1      = X(1)                                                      INT06480
      XN      = X(N)                                                      INT06490
      CH      = XN -X1                                                    INT06500
      FN1     = FLOAT(N-1)                                                INT06510
      N10      = N/10                                                     INT06520
      DO 20 I=1,N                                                         INT06530
      T(I)    = FLOAT(I-1)/FN1                                            INT06540
      X(I)    = (X(I)-X1)/CH                                              INT06550
   20 CONTINUE                                                            INT06560
C     CALL SMFIT(N10,N,T,X,D1,N10)                                        INT06570
C     IF(X(2).LT.0.35 * X(3)) X(2) = 0.35 * X(3)                          INT06580
C     CALL SMFIT(1,N,T,X,D1,2)                                            INT06590
      CALL AMEAN(1,N,T,X,N10)                                             INT06600
C                                                                         INT06610
      RETURN                                                              INT06620
      END                                                                 INT06630
C          DATA SET KCBCBL2D   AT LEVEL 001 AS OF 08/24/84                INT06640
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```
C          DATA SET KCBCBL2D   AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCBL2D   AT LEVEL 012 AS OF 04/06/84
      SUBROUTINE BL2D ( ITR,ISWPT,SURFID)
C    PROGRAM CALCULATES VISCOUS/INVISCID INTERACTION USING HILBERT
C    INTEGRAL.
C
      COMMON /BLC0/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON /BLC1/ F(101,2),U(101,2),V(101,2),W(101,2),B(101,2)
      COMMON /BLC2/ DELF(101),DELU(101),DELV(101),DELW(101)
      COMMON /BLC7/ C(100,100),D(100),DB(100),DBP(100),UE0(100),GI
      COMMON/EDDY1/ RL,RX,SQRX,RXNTR,GMTR,GMTRS(100),ALFAS(100),
     +              FFS(100),RTS(100),IEDY,NXSPT
      COMMON /SMRY/ VW(100),ITP(100),ISL(100),DLS(100),CF(100),
     +              THT(100),NPSTR(100)
      COMMON /GTY / X(101),UE(100),P1(100),P2(100),CEL,CELH
      COMMON /BONV/ ITMAX,EPSL,EPST,CONV
      COMMON /SAVE/ FS(101),US(101),VS(101),WS(101),BS(101)
      COMMON /BLIN/ TITLE(20),XC(100),YC(100),ISG(100),DELS(100),
     +             XCTR,XTR,ISTRP,ICYCLE,ICYTL,XCTRS(2),TRFIND(2)
      COMMON /ISURF/ ISF
      COMMON/PLOT/NVP(2),NXVP(20,2),ICC
      DIMENSION SURFID(4)
C
C     ------------------------------------------------------------
C
C     GENERATE B. L. GRIDS + SET INITIAL CONDITIONS
C
      DO 5 I = 1 , NXT
      ALFAS(I) = 0.0
      FFS(I)   = 1.0
      RTS(I)   = 1.0
5     CONTINUE
      CALL INPUT(ITR,ISWPT,SURFID)
C
C     CALCULATE HILBERT COEFFS., C(I,J)
C
      CALL CALCIJ(NXT,0)
C
C    LOOP OF CALCULATIONS
C
      NSS   = NS
      NXSPT = NXT + 1
C     IF ( ICYCLE .EQ. 1 ) NS = NXT + 1
10    NX    = NX + 1
      CEL   = 0.5 * (X(NX) + X(NX-1)) /(X(NX) -X(NX-1))
      CELH  = 0.5 * CEL
20    IT    = 0
      RX    = UE(NX)*X(NX)*RL
      SQRX  = SQRT(RX)
30    IT    = IT + 1
      IF(IT .LE. ITMAX) GO TO 40
      NXM1  = NX-1
      CALL HEADER( TITLE,SURFID,ISTRP )
      WRITE(6, 170 ) (M,X(M),CF(M),DLS(M),UE(M),P2  (M),THT(M),
     +             D(M),ALFAS(M),  ITP(M),NPSTR(M),M=1,NXM1)
      WRITE(6, 160 ) NX
```

```
      STOP
 40   CONTINUE
      IF(NX .GT. NTR) CALL EDDY
      CALL COEFTR
      CALL SOLV3
      IF(V(1,2).GT.0.0) GOTO 60
C
C     EXTRAPOLATE CALCULATED D FOR TURBULENT SEPARATION OR LAMINAR
C     SEPARATION FOR LAMINAR FLOW CALCULATION ONLY
C
      CALL EXTRAP(NX,NXT,X,D)
      NXM1 = NX - 1
      CALL HEADER( TITLE,SURFID,ISTRP )
      WRITE(6, 170 ) (M,X(M),CF(M),DLS(M),UE(M),P2(M),THT(M),
     +               D(M),ALFAS(M),ITP(M),NPSTR(M),M=1,NXM1)
      WRITE(6,180)
      WRITE(6,190) (M,X(M),D(M),M=NX,NXT)
      GOTO 130
 60   IF(NX .GT. NTR) GO TO 70
      IF(ABS(DELV(1)) .GT. EPSL) GO TO 30
      GO TO 80
 70   CONTINUE
      IF(ABS(DELV(1)/V(1,2)) .GT. EPST) GO TO 30
 80   CONTINUE
C
C     CHECK FOR GROWTH
C
      IF(NP .GE. NPT) GO TO 90
      IF(ABS(V(NP,2)) .LT. 0.0005 .AND. ABS(1.0-U(NP-2,2))
     +        .LT.0.0035) GOTO 90
      CALL FILLUP(1)
      IT = 1
      GO TO 30
 90   CONTINUE
C
      CALL FILLUP(2)
      CALL OUTPUT(1)
      IF(ITR.EQ.0 .OR. NX.GE.NTR) GOTO 100
      IF(NX.LT.3 .OR. ITR.NE.3) GO TO 100
      CALL TRNS(ICODE)
      IF(ICODE.EQ.1) GOTO 20
 100  IF(NX .NE. NSS) GOTO 120
C
C   STORE PROFILES AT THE STATION NS FOR INVERSE B. L.
C   CALCULATION
C
      DO 110 J = 1 , NPT
      FS(J) = F(J,2)
      US(J) = U(J,2)
      VS(J) = V(J,2)
      WS(J) = W(J,2)
      BS(J) = B(J,2)
 110  CONTINUE
 120  IF ( NX .LT. NSS ) GOTO 10
C     IF (ICYCLE .NE. 1) GO TO 130
      IF ( NX .GE. NS )  GOTO 130
```

```
      IF (NX .LT. NXT) GO TO 10
      CALL HEADER( TITLE,SURFID,ISTRP )
      WRITE(6, 170 ) (M,X(M),CF(M),DLS(M),UE(M),P2  (M),THT(M),
     +             D(M), ALFAS(M), ITP(M),NPSTR(M),M=1,NXT)
130   DO 140 I = 1 , NXT
140   DB(I) = D(I)
      NS = NSS
      NX = NS
      NP = NPSTR(NX)
      DO 150 J = 1 , NPT
      F(J,2) = FS(J)
      U(J,2) = US(J)
      V(J,2) = VS(J)
      W(J,2) = WS(J)
      B(J,2) = BS(J)
150   CONTINUE
155   INVRS = NS + 1
C
C   CALCULATION SHIFTS TO USING PHYSICAL COORDINATES
      CALL MAIN2(ITR,ISWPT,SURFID)
C
C     PASS DELTA-STAR BACK TO MAIN PROG.
C
      DO 158 I = 1,NXT
      DELS(I) = DLS(I)
158   CONTINUE
      RETURN
C
C   -----------------------------------------------------------------
C
160   FORMAT(1H0,' ** ITERATIONS EXCEEDED ITMAX AT NX = ',I5/
     +            1H ,' ** CALCULATIONS STOP. ** ')
170   FORMAT(1H0,' ** SUMMARY OF STANDARD B. L. SOLUTIONS. **'/
     +            1H0,4X,2HNX,7X,1HX,12X,2HCF,11X,3HDLS,12X,2HUE,
     +            12X,2HP2,11X,3HTHT,13X,1HD,10X,4HALFA,6X,2HIT,2X,2HNP/
     +            (1H ,3X,I3,F10.5,2X,7E14.5,2I4))
180   FORMAT(1H0,34H FLOW SEPARATES. D IS EXTRAPOLATED/
     +            1H0,3X,3H NX,7X,1HX,13X,1HD/)
190   FORMAT(1H ,3X I3,F10.5,2X,E14.5)
      END
C          DATA SET KCBCCALCIJ AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCCALCIJ AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCCALCIJ AT LEVEL 005 AS OF 04/05/84
      SUBROUTINE CALCIJ ( IL, LO)
C
C   CALCULATE HILBERT INTEGRAL COEFFS
C
      COMMON /BLC7/ C(100,100),D(100),DB(100),DBP(100),UE0(100),GI
      COMMON/EDDY1/ RL,RX,SQRX,RXNTR,GMTR,GMTRS(100)
     +                ,ALFAS(100),FFS(100),RTS(100),IEDY,NXSPT
      COMMON /GTY / X(101),UE(100),P1(100),P2(100),CEL,CELH
      DIMENSION    E(2)
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
      PI      = 3.14159265
```

```
      PI       = PI*SQRT(RL)
      IL1      = IL  - 1
      DO 65 I = 2, IL1
      E (1)  = 0.
      L      = L0 + I
      DO 60 J = 2, IL
      J1       = J - 1
      K        = J + L0
      DX1      = X(L) - X(K)
      DX2      = X(K) - X(K-1)
      DX3      = X(L) - X(K-1)
      IF ( J .EQ. I ) GO TO 30
      IF ( J .EQ. (I+1) ) GO TO 40
C
C     J .NE. I OR I+1
C
      E (2)    = ( 1.0/DX2 ) * ALOG( ABS( DX3 / DX1 ) )
      GO TO 50
C
C     J .EQ. I
C
30    R1       = ( X(K+1)-X(K) ) / (X(K+1)-X(K-1) )
      E (2)    = ( R1 * ALOG( ABS( DX3 / ( X(L)-X(K+1)) ) ) + 2.0 ) / DX2
      GO TO 50
C
C     J .EQ. I+1
C
40    R1       = ( X(K-1)-X(K-2) ) / ( X(K)-X(K-2) )
      E (2)    = ( R1 * ALOG( ABS( (X(L)-X(K-2)) / DX1 ) ) - 2.0 ) / DX2
C
50    CONTINUE
      C (J1,I)= ( E(1) - E(2) ) / PI
      E(1)     = E (2)
60    CONTINUE
      E (2)    = 0.
      J1       = IL
      K        = K + 1
      C (J1,I)= E(1) / PI
65    CONTINUE
C
C
      RETURN
      END
C          DATA SET KCBCCOEF   AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCCOEF   AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCCOEF   AT LEVEL 007 AS OF 04/05/84
      SUBROUTINE COEF(GAMMA1,GAMMA2)
C
C    CALCULATE COEFFS OF B. L. FINITE-DIFFERENCE EQS. IN
C    SEMI-TRANSF VARIABLES( AFTER SWITCHING).
C
      COMMON /BLC0/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON /BLC1/ F(101,2),U(101,2),V(101,2),W(101,2),B(101,2)
      COMMON /BLC6/ S1(101),S2(101),S3(101),S4(101),S5(101),S6(101),
     +              S7(101),S8(101),R1(101),R2(101),R3(101),R4(101)
      COMMON /BLC7/ C(100,100),D(100),DB(100),DBP(100),UE0(100),GI
```

```
      COMMON /GRD / ETA(101),DETA(101),A(101)
      COMMON /GTY / X(101),UE(100),P1(100),P2(100),CEL,CELH
C
C   ------------------------------------------------------------
C
      P1H = 0.5 * P1(NX)
      DO 100 J= 2,NP
      FLARE  = 1.0
      FB     = 0.5*(F(J,2) + F(J-1,2))
      UB     = 0.5*(U(J,2) + U(J-1,2))
      FVB    = 0.5*(F(J,2)*V(J,2)+F(J-1,2)*V(J-1,2))
      IF(UB .LT. 0.0) FLARE = 0.0
      VB     = 0.5*(V(J,2) + V(J-1,2))
      USB    = 0.5*(U(J,2)**2 + U(J-1,2)**2)
      WSB    = 0.5*(W(J,2)**2 + W(J-1,2)**2)
      DERBV  =(B(J,2)*V(J,2) - B(J-1,2)*V(J-1,2))/DETA(J-1)
      FB4    = 0.5*(F(J,1) + F(J-1,1))
      VB4    = 0.5*(V(J,1) + V(J-1,1))
      USB4   = 0.5*(U(J,1)**2 + U(J-1,1)**2)
      WSB4   = 0.5*(W(J,1)**2 + W(J-1,1)**2)
      FVB4   = 0.5*(F(J,1)*V(J,1)+F(J-1,1)*V(J-1,1))
      DERBV4 = (B(J,1)*V(J,1) - B(J-1,1)*V(J-1,1))/DETA(J-1)
      S1(J)  = CELH*(FB - FB4) + P1H*F(J,2) + B(J,2)/DETA(J-1)
      S2(J)  = CELH*(FB - FB4) + P1H*F(J-1,2) - B(J-1,2)/DETA(J-1)
      S3(J)  = CELH*(VB + VB4) + P1H*V(J,2)
      S4(J)  = CELH*(VB + VB4) + P1H*V(J-1,2)
      S5(J)  = -CEL*FLARE*U(J,2)
      S6(J)  = -CEL*FLARE*U(J-1,2)
      S7(J)  = CEL*W(J,2)
      S8(J)  = CEL*W(J-1,2)
C
      CRB    = -DERBV4 + CEL*WSB4 - CEL*FLARE*USB4 - P1(NX)*FVB4
      R2(J)  = CRB - (DERBV - CEL*FLARE*USB  + CEL*(VB+VB4)*(FB-FB4) +
     +                    CEL*WSB + P1(NX)*FVB)
      R1(J)  = F(J-1,2) - F(J,2) + DETA(J-1)*UB
      R3(J-1)= U(J-1,2) - U(J,2) + DETA(J-1)*VB
      R4(J-1)= W(J-1,2) - W(J,2)
  100 CONTINUE
C
C   BOUNDARY CONDITIONS
C
      R1(1)  = 0.0
      R2(1)  = 0.0
      R4(NP) = 0.0
      IF(NX .GE. INVRS) GO TO 120
      GAMMA1 = 0.0
      GAMMA2 = 1.0
      R3(NP) = 0.0
      RETURN
  120 CONTINUE
      CII    = C(NX,NX) * SQRT(X(NX))
      GAMMA1 = 1.0
      GAMMA2 = (1.0 - CII*ETA(NP))/CII
      R3(NP) = (GI + CII*(ETA(NP)*W(NP,2) - F(NP,2)) -W(NP,2))/CII
C
      RETURN
```

```
      END
C          DATA SET KCBCCOEFTR AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCCOEFTR AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCCOEFTR AT LEVEL 004 AS OF 02/21/84
      SUBROUTINE COEFTR
C
C   CALCULATE COEFFS. OF B. L. FINITE-DIFFERENCE EQS.
C   IN TRANSFORMED VARIABLES( BEFORE SWITCHING).
C
      COMMON /BLC0/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON /BLC1/ F(101,2),U(101,2),V(101,2),W(101,2),B(101,2)
      COMMON /BLC6/ S1(101),S2(101),S3(101),S4(101),S5(101),S6(101),
     +              S7(101),S8(101),R1(101),R2(101),R3(101),R4(101)
      COMMON /GRD / ETA(101),DETA(101),A(101)
      COMMON /GTY / X(101),UE(100),P1(100),P2(100),CEL,CELH
C
C   ---------------------------------------------------
C
C
      DO 100 J= 2,NP
      FB     = 0.5*(F(J,2) + F(J-1,2))
      UB     = 0.5*(U(J,2) + U(J-1,2))
      VB     = 0.5*(V(J,2) + V(J-1,2))
      USB    = 0.5*(U(J,2)**2 + U(J-1,2)**2)
      DERBV = (B(J,2)*V(J,2) - B(J-1,2)*V(J-1,2))/DETA(J-1)
      FVB = 0.5*(F(J,2)*V(J,2) + F(J-1,2)*V(J-1,2))
      FVB4 = 0.5*(F(J,1)*V(J,1) + F(J-1,1)*V(J-1,1))
      FB4 = 0.5*(F(J,1) + F(J-1,1))
      VB4 = 0.5*(V(J,1) + V(J-1,1))
      USB4   = 0.5*(U(J,1)**2 + U(J-1,1)**2)
      DERBV4 = (B(J,1)*V(J,1) - B(J-1,1)*V(J-1,1))/DETA(J-1)
C
      S1(J) = CELH*(FB-FB4) + 0.5*P1(NX)*F(J,2) + B(J,2)/DETA(J-1)
      S2(J) = CELH*(FB-FB4) + 0.5*P1(NX)*F(J-1,2) -B(J-1,2)/
     +            DETA(J-1)
      S3(J) = CELH*(VB + VB4) + 0.5*P1(NX)*V(J,2)
      S4(J) = CELH*(VB + VB4) + 0.5*P1(NX)*V(J-1,2)
      S5(J) = -(CEL+P2(NX))*U(J,2)
      S6(J) = -(CEL+P2(NX))*U(J-1,2)
C
      CLB = DERBV4  + P1(NX-1)*FVB4 - P2(NX-1)*USB4 + P2(NX-1)
      CRB = -CLB - CEL*USB4 - P2(NX)
      R2(J) = CRB - (DERBV + P1(NX)*FVB- (CEL+P2(NX))*USB  + CEL*
     +            (VB + VB4)*(FB - FB4))
C
      R1(J)= F(J-1,2) - F(J,2) + DETA(J-1)*UB
      R3(J-1)= U(J-1,2) - U(J,2) + DETA(J-1)*VB
100   CONTINUE
      R1(1)  = 0.0
      R2(1)  = 0.0
      R3(NP) = 0.0
      RETURN
      END
C          DATA SET KCBCCOMPBL AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCCOMPBL AT LEVEL 001 AS OF 08/24/84
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```
C         DATA SET KCBCCOMPBL AT LEVEL 010 AS OF 08/24/84
C
      SUBROUTINE COMPBL(CASEID,XP,YP,VT,S,DLSP,DLS,DBPP,NBL,ITRI,XCTRI,
     +                         RN,NT,ISWPT)
C
      COMMON /BLCO/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON /BLC7/ C(100,100),D(100),DB(100),DBP(100),UEO(100),GI
      COMMON/EDDY1/ RL,RX,SQRX,RXNTR,GMTR,GMTRS(100)
     +              ,ALFAS(100),FFS(100),RTS(100),IEDY,NXSPT
      COMMON /GTY / X(101),UE(100),P1(100),P2(100),CEL,CELH
      COMMON /BLIN/ TITLE(20),XC(100),YC(100),ISG(100),DELS(100),
     +           XCTR,XTR,ISTRP,ICYCLE,ICYTL,XCTRS(2),TRFIND(2)
      COMMON /BLOW/ VN(100)
      COMMON /ISURF/ ISF
      COMMON/PLOT/NVP(2),NXVP(20,2),ICC
C
      DIMENSION     XP(100),DLSP(100),YP(100),VT(100),S(100),
     +              DBPP(100),DLS(100),CASEID(20)
      DIMENSION     XIN(100,2),YIN(100,2),SI(100,2),VIN(100,2),
     +                   DIN(100,2),DELSTR(100,2),DD(100,2),DDD(100,2)
      DIMENSION     XB(101),D1(101),D2(101),D3(101),IEND(2)
      DIMENSION     NBL(2),ITRI(2),XCTRI(2)
      LOGICAL TRFIND
      CHARACTER * 4 SURF(4),STITLE(2),SURFID(4)
C
      DATA     SURF   / '    ','R SU','RFAC','E   ' /
      DATA     STITLE / 'UPPE','LOWE' /
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
90    FORMAT ( 1H1,5X,'COMPUTING BOUNDARY LAYER USING HILBERT',
     +               ' INTEGRAL',/)
110   FORMAT ( 1H0,6X,'I',9X,'XP',13X,'YP',13X,'S',14X,'VT',13X,
     +              'DBP' / )
112   FORMAT ( 1H0,6X,'I',4X,'II',3X,'IK',7X,'XIN',12X,'YIN',
     +              13X,'SI',12X,'VIN',12X,'DIN' / )
120   FORMAT ( 1H ,5X,I3,5E15.6 )
122   FORMAT ( 1H ,3X,3(2X,I3),5E15.6 )
130   FORMAT ( 1H0,5X,'STAGNATION POINT IS FOUND BETWEEN POINT NO. ',
     +              I3,' AND POINT NO. ',I3 / )
140   FORMAT ( 1H0,5X,'INTERPOLATED STAGNATION POINT VALUES' /
     +              1H0,5X,'S = ',E13.6,2X,'XP = ',E13.6,2X,'YP = ',
     +              E13.6,2X,'DBP = ',E13.6,2X,'VT = 0.0' / )
150   FORMAT ( 1H0,5X,'TOTAL NUMBER OF UPPER SURFACE POINTS = ',I5,
     +              2X,'AND AT LOWER SURFACE = ',I5 / )
160   FORMAT ( 1H0,5X,'UPPER SURFACE DATA' )
170   FORMAT ( 1H0,5X,'LOWER SURFACE DATA' )
180   FORMAT ( 1H0,5X,'UPPER SURFACE CALCULATIONS' / )
182   FORMAT ( 1H0,5X,'LOWER SURFACE CALCULATIONS' / )
190   FORMAT ( 1H0,5X,'RESULTS OF POINT REDISTRIBUTION' / )
200   FORMAT ( 1H0,5X,'TABLE OF DELTA-STARS' / ( 1H ,5X,8E15.6 ) )
220   FORMAT (1H0,5X,'TABLE OF BLOWING-VEL.' / (1H ,5X,8E15.6) )
210   FORMAT ( 1H0,5X,'NO CHANGE OF SIGN ON VT. CANNOT FIND STAG. PT.' )
C
C             -------------- * -------------
C
```

```
C                                                                               INT10560
C     READ ONE STRIP INPUT DATA FROM UNIT NO. 1.  THE ORDER IS                  INT10570
C     FROM THE LOWER SURFACE T.E. TO THE UPPER SURFACE T.E.                     INT10580
C                                                                               INT10590
      DO 230 I = 1,20                                                           INT10600
      TITLE(I) = CASEID(I)                                                      INT10610
230   CONTINUE                                                                  INT10620
      RL     = RN                                                               INT10630
      DO 300 I = 1,NT                                                           INT10640
      DBP(I) = DBPP(I)                                                          INT10650
300   CONTINUE                                                                  INT10660
C                                                                               INT10670
C     PRINT THE INPUT DATA.                                                     INT10680
C                                                                               INT10690
      P2(1)  = 1.0                                                              INT10700
C     WRITE ( 6,90 )                                                            INT10710
C     WRITE ( 6,110 )                                                           INT10720
C     DO 500 I = 1,NT                                                           INT10730
C     WRITE ( 6,120 ) I,XP(I),YP(I),S(I),VT(I),DBP(I)                           INT10740
C500  CONTINUE                                                                  INT10750
C                                                                               INT10760
C     FIND STAGNATION POINT                                                     INT10770
C                                                                               INT10780
      DO 600 I = 2,NT                                                           INT10790
      VPROD  = VT(I) * VT(I-1)                                                  INT10800
      IF ( VPROD .GT. 0. ) GO TO 600                                            INT10810
      IS     = I                                                                INT10820
      IS1    = IS - 1                                                           INT10830
      GO TO 700                                                                 INT10840
600   CONTINUE                                                                  INT10850
      WRITE ( 6,210 )                                                           INT10860
      STOP 1                                                                    INT10870
C                                                                               INT10880
C                                                                               INT10890
C     INTERPOLATE S AT VT = 0.                                                  INT10900
C                                                                               INT10910
C     WRITE ( 6,130 ) IS1,IS                                                    INT10920
700   DS     = S(IS) - S(IS1)                                                   INT10930
      DV     = VT(IS) - VT(IS1)                                                 INT10940
      SS     = S(IS) - VT(IS) * ( DS/DV )                                       INT10950
      DBB    = DBP(IS) - DBP(IS1)                                               INT10960
      DX     = XP(IS) - XP(IS1)                                                 INT10970
      DY     = YP(IS) - YP(IS1)                                                 INT10980
      DS1    = SS - S(IS)                                                       INT10990
      DBS    = DBP(IS) + DS1 * ( DBB/DS )                                       INT11000
      XPS    = XP(IS)  + DS1 * ( DX /DS )                                       INT11010
      YPS    = YP(IS)  + DS1 * ( DY /DS )                                       INT11020
C     WRITE ( 6,140 ) SS,XPS,YPS,DBS                                            INT11030
C                                                                               INT11040
C     IU IS THE TOTAL UPPER SURFACE POINTS. + STAG. PT.                         INT11050
C     IL IS THE TOTAL LOWER SURFACE POINTS  + STAG. PT.                         INT11060
C                                                                               INT11070
      IU     = NT - IS + 2                                                      INT11080
      IL     = IS                                                               INT11090
C     WRITE ( 6,150 ) IU,IL                                                     INT11100
C                                                                               INT11110
```

```
C     GROUP THE DATA FOR EACH SURFACE. FIRST, UPPER.
C
      DO 1200 L = 1,2
      GO TO ( 800,900 ), L
C
C     L = 1 IS UPPER SURFACE
C     L = 2 IS LOWER SURFACE
C
800   M1      = IS
      M2      = NT
      IEND(L) = IU
      GO TO 1000
900   M1      = 1
      M2      = IL-1
      IEND(L) = IL
C
1000  I = 1
      XIN(I,L) = XPS
      YIN(I,L) = YPS
      SI (I,L) = 0.
      DIN(I,L) = DBS
      VIN(I,L) = 0.
      IF ( IP .GE. 1 ) THEN
      IF ( L .EQ. 1 ) WRITE ( 6,160 )
      IF ( L .EQ. 2 ) WRITE ( 6,170 )
      WRITE ( 6,112 )
      WRITE ( 6,122 ) I,I,I,XIN(1,L),YIN(1,L),SI(1,L),VIN(1,L),
     +                DIN(1,L)
      END IF
      DO 1100 II = M1,M2
      I       = I + 1
      IK      = II
      IF ( L .EQ. 2 ) IK = IL - II
      XIN(I,L) = XP(IK)
      YIN(I,L) = YP(IK)
      SI (I,L) = S(IK)-SS
      IF ( L .EQ. 2 ) SI(I,L) = SS-S(IK)
      VIN(I,L) = ABS(VT(IK))
      DIN(I,L) = DBP(IK)
      IF(IP .GE. 1)WRITE ( 6,122 ) I,II,IK,XIN(I,L),YIN(I,L),SI(I,L),
     +               VIN(I,L),DIN(I,L)
1100  CONTINUE
C
1200  CONTINUE
C
C     RE-DISTRIBUTE POINTS ON EACH SURFACE TO A DENSER NUMBER.
C
C     WRITE ( 6,90 )
      DO 2000 ISF = 1,2
      NN      = IEND(ISF)
      ITR     = ITRI(ISF)
      NXT     = NBL(ISF)
      XCTR    = XCTRI(ISF)
      SURF  (1) = STITLE(ISF)
      ICC     = 1
      DO 1220 J = 1,4
```

```
      SURFID(J) = SURF(J)
 1220 CONTINUE
C     IF ( ISF .EQ. 1 ) WRITE ( 6,180 )
C     IF ( ISF .EQ. 2 ) WRITE ( 6,182 )
      SCALE  = SI(NN,ISF)
C
      CALL BLGRID ( NXT,XB,D1,D2 )
C
      DO 1300 I = 1,NXT
 1300 X (I)  = XB(I) * SCALE
C
C     INTERPOLATE S,VT,X,Y,D INTO THE NEW DISTRIBUTION.
C
      CALL SMFIT ( 1,NN,SI(1,ISF),VIN(1,ISF),D1,2 )
      CALL SMFIT ( 1,NN,SI(1,ISF),DIN(1,ISF),D1,2 )
      CALL DIFF3 ( NN,SI(1,ISF),VIN(1,ISF),D1,D2,D3,0 )
      CALL INTRP3( NN,SI(1,ISF),VIN(1,ISF),D1,D2,D3,NXT,X,UE )
      CALL DIFF3 ( NN,SI(1,ISF),DIN(1,ISF),D1,D2,D3,0 )
      CALL INTRP3( NN,SI(1,ISF),DIN(1,ISF),D1,D2,D3,NXT,X,DB )
      CALL DIFF3 ( NN,SI(1,ISF),XIN(1,ISF),D1,D2,D3,0 )
      CALL INTRP3( NN,SI(1,ISF),XIN(1,ISF),D1,D2,D3,NXT,X,XC )
      CALL DIFF3 ( NN,SI(1,ISF),YIN(1,ISF),D1,D2,D3,0 )
      CALL INTRP3( NN,SI(1,ISF),YIN(1,ISF),D1,D2,D3,NXT,X,YC )
      IF ( IP .GE. 1 ) THEN
      WRITE ( 6,190 )
      WRITE ( 6,110 )
      DO 1320 I = 1,NXT
      WRITE ( 6,120 ) I,XC(I),YC(I),X(I),UE(I),DB(I)
 1320 CONTINUE
      END IF
C
C     INPUT TO THE B.L. PROGRAM X,UE,DB,XC,YC ARE NOW DEFINED.
C
      DO 1350 I = 1,NXT
      DBP(I) = DB(I)
      D(I)   = DB(I)
 1350 CONTINUE
C
      CALL BL2D( ITR,ISWPT,SURFID)
C
      CALL DIFF3 ( NXT,X,DELS,D1,D2,D3,0 )
      CALL INTRP3( NXT,X,DELS,D1,D2,D3,NN,SI(1,ISF),DELSTR(1,ISF) )
      CALL DIFF3(NXT,X,D,D1,D2,D3,0)
      CALL INTRP3(NXT,X,D,D1,D2,D3,NN,SI(1,ISF),DD(1,ISF))
      CALL DIFF3(NN,SI(1,ISF),DD(1,ISF),DDD(1,ISF),D2,D3,0)
      TRFIND(ISF) = .FALSE.
      IF(ITR .EQ. 3 .AND. NTR .LE. NXT) THEN
      XCTRS(ISF) = XCTR
      TRFIND(ISF) = .TRUE.
      END IF
C
 2000 CONTINUE
C
C     PUT THE TWO SURFACES DELTA-STARS BACK TO ONE STRIP
C
      DELSTR(1,2) = 0.5*(DELSTR(2,1)+DELSTR(2,2))
```

```
      DELSTR(1,1) = DELSTR(1,2)
      DD(1,2)  = 0.5 * (DD(2,1) + DD(2,2))
      DD(1,1)  = DD(1,2)
      DDD(1,2) = 0.5 * (DDD(2,1) + DDD(2,2)) /SQRT(RL)
      DDD(1,1) = DDD(1,2)
      IL = IEND(2)
      I      = IL
      DO 2100 II = 2,IL
      I      = I-1
      DLS(I) = DELSTR(II,2)
      VN(I) = DDD(II,2)/SQRT(RL)
      DBPP(I) = DD(II,2)
2100  CONTINUE
C
      I       = IL-1
      IU      = IEND(1)
      DO 2200 II=2,IU
      I       = I + 1
      DLS(I) = DELSTR(II,1)
      VN(I) = DDD(II,1)/SQRT(RL)
      DBPP(I) = DD(II,1)
2200  CONTINUE
C     WRITE ( 6,200 ) ( DLS(I), I=1,NT )
C     WRITE ( 6,220) (VN(I) , I=1,NT)
C
      RETURN
      END
C          DATA SET KCBCCOMPGI AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCCOMPGI AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCCOMPGI AT LEVEL 003 AS OF 08/24/84
      SUBROUTINE COMPGI
C
C   CALCULATE GI
C
      COMMON /BLC0/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON /BLC7/ C(100,100),D(100),DB(100),DBP(100),UE0(100),GI
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
      SUM2      = 0.0
      N1        = NX + 1
      N2        = NXT
      DO 180 K = N1,N2
      SUM2      = SUM2 + C(K,NX)* (D(K) - DBP(K))
180   CONTINUE
C
      N1        = 2
      N2        = NX-1
      SUM1      = 0.
      DO 260 K = N1,N2
      SUM1      = SUM1 + C(K,NX)* (D(K) - DBP(K))
260   CONTINUE
      GI        = UE0(NX) + SUM1 + SUM2 - C(NX,NX) * DBP(NX)
C
      RETURN
```

```
      END
C         DATA SET KCBCDIFF3  AT LEVEL 001 AS OF 08/24/84
C         DATA SET KCBCDIFF3  AT LEVEL 001 AS OF 08/24/84
C         DATA SET KCBCDIFF3  AT LEVEL 002 AS OF 04/05/84
      SUBROUTINE DIFF3 (N,X,F,FP,FPP,FPPP,IEND)
C
C     DETERMINES THE DERIVATIVE OF THE INPUT FUNCTION AT THE INPUT PTS.
C
      DIMENSION X(101),F(101),FP(101),FPP(101),FPPP(101)
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C
C     FIRST DERIVATIVES USING WEIGHTED ANGLES
C
      N1=N-1
      DX=X(2)-X(1)
      DF=F(2)-F(1)
      ANG2= ATAN2(DF,DX)
      DL2=DX
C
      DO 10 I=2,N1
      ANG1=ANG2
      DL1=DL2
      I1=I+1
      DX=X(I1)-X(I)
      DF=F(I1)-F(I)
      ANG2= ATAN2(DF,DX)
      DL2=DX
      ANG=(DL2*ANG1+DL1*ANG2)/(DL1+DL2)
      FP(I)= TAN(ANG)
C
      IF (I.NE.2) GO TO 10
      ANGI = ANG
      ANG1I = ANG1
      DLI = DL1
C
10    CONTINUE
      ANGF = ANG
      ANG2F = ANG2
      DLF = DL2
      IEND1 = IEND + 1
      GO TO (11,12,13), IEND1
C
C     IEND = 0 , EXTRAPOLATE FOR END VALUES
C
11    FP(1) = 2.*(F(2)-F(1 ))/DLI - FP(2)
      FP(N) = 2.*(F(N)-F(N1))/DLF - FP(N1)
      GO TO 20
C
C     IEND = 1, DERIVATIVES ARE CONTINUOUS ACROSS ENDS
C
12    ANG = (DLI*ANG2F + DLF*ANG1I) / (DLI + DLF)
      FP(1) =  TAN(ANG)
      FP(N) = FP(1)
```

```
      GO TO 20
C
C  IEND = 2, IF FIRST DERIVATIVE .LT. 0.0 RESET TO ZERO
C
13    CONTINUE
      FP(1) = 2.*(F(2)-F(1 ))/DLI - FP(2)
      IF (FP (1) .LT. 0.0) FP (1) = 0.0
      FP(N) = 2.*(F(N)-F(N1))/DLF - FP(N1)
C
C     SECOND + THIRD DERIVATIVES USING CUBIC FITS
C
20    DO 30 I=2,N1
      I1      = I - 1
      I2      = I + 1
      DX1     = X (I1) - X (I)
      DX2     = X (I2) - X (I)
      DF1     = 2.0 * ((F (I1) - F (I)) / DX1 - FP (I)) / DX1
      DF2     = 2.0 * ((F (I2) - F (I)) / DX2 - FP (I)) / DX2
      FPPP(I)= 3.0 * (DF1 - DF2) / (DX1 - DX2)
      FPP (I)= DF1 - DX1 * FPPP (I) / 3.0
30    CONTINUE
      FPPP(1)= FPPP (2)
      FPPP(N)= FPPP (N1)
      FPP (1)= FPP (2 ) + (X (1) - X (2 )) * FPPP (2 )
      FPP (N)= FPP (N1) + (X (N) - X (N1)) * FPPP (N1)
C
      RETURN
      END
C          DATA SET KCBCEDDY   AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCEDDY   AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCEDDY   AT LEVEL 003 AS OF 04/05/84
      SUBROUTINE EDDY
C
C   CALCULATE EDDY VISCOSITY USING C .S. TWO-LAYERS EDDY
C   VISCOSITY FORMULA
      COMMON /BLC0/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON /BLC1/ F(101,2),U(101,2),V(101,2),W(101,2),B(101,2)
      COMMON/BLC7/ C(100,100),D(100),DB(100),DBP(100),UE0(100),GI
      COMMON/EDDY1/ RL,RX,SQRX,RXNTR,GMTR,GMTRS(100),
     +                    ALFAS(100),FFS(100),RTS(100),IEDY,NXSPT
      COMMON /GRD / ETA(101),DETA(101),A(101)
      COMMON /GTY / X(101),UE(100),P1(100),P2(100),CEL,CELH
      DIMENSION FINT(101)
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
      JO=1
      UDEL=0.995*U(NP,2)
      DO 10 J=1,NP
      IF(U(J,2).LT.UDEL) JJ=J
10    IF(U(J,2).LT.0.0)  JO=J
      EDEL=ETA(JJ)+(ETA(JJ+1)-ETA(JJ))/(U(JJ+1,2)-U(JJ,2))
     +     *(UDEL-U(JJ,2))
      DO 15 J=1,NP
      ETADEL=ETA(J)/EDEL
```

```
      IF(ETADEL.GT.1.0) ETADEL=1.0                                      INT13890
15    FINT(J)=1.0/(1.0+5.5*ETADEL**6)                                   INT13900
      CALL AMEAN(1,JJ,ETA,FINT,2)                                       INT13910
      RU = RL                                                           INT13920
      IF (IT .GT. 1) GO TO 20                                           INT13930
      ALFAS(NX) = ALFAS(NX-1)                                           INT13940
      FFS(NX) = FFS(NX-1)                                               INT13950
      RTS(NX) = RTS(NX-1)                                               INT13960
C                                                                       INT13970
      GMTR   = GMTRS(NX)                                                INT13980
      IF (NX .LE. NS) RU = RL * UE(NX)                                  INT13990
      RL2    = SQRT(RU * X(NX))                                         INT14000
      RL4    = SQRT(RL2)                                                INT14010
      RL216  = 0.16 * RL2                                               INT14020
20    VMAX   = 0.5 * (V(1,2) + V(1,1))                                  INT14030
      DO 30 J=2,NP                                                      INT14040
      VM     = 0.5 * (V(J,2)+V(J,1))                                    INT14050
      IF(VM .GT. VMAX) VMAX = VM                                        INT14060
30    CONTINUE                                                          INT14070
      IF (IEDY .EQ. 0) GO TO 35                                         INT14080
      IF (IT .LE. 1 .OR. GMTR .LT. 0.85 .OR. NX .LE. NTR+3 )            INT14090
     +     GO TO 35                                                     INT14100
C                                                                       INT14110
C   MODIFY ALFA USING SIMPSON'S ARGUMENTS                               INT14120
C     CALL SMPSON                                                       INT14130
35    ALFA = ALFAS(NX)                                                  INT14140
      EDVO   = ALFA * RL2 * GMTR * (U(NP,2)*ETA(NP) - F(NP,2))          INT14150
      DO 40 J=2,NP                                                      INT14160
      JJ     = J                                                        INT14170
      YBA    = RL4*SQRT(VMAX)/26.0*ETA(J)                               INT14180
      EL     = 1.0                                                      INT14190
      IF(YBA .LT. 10.0) EL = 1.0 - EXP(-YBA)                            INT14200
      EDVI   = RL216 * GMTR * (EL*ETA(J))**2 * ABS(V(J,2))              INT14210
      IF(EDVI .GT. EDVO) GO TO 70                                       INT14220
      B(J,2) = 1.0 + EDVI*FINT(J)                                       INT14230
      IF(B(J,2) .LT. B(J-1,2)) B(J,2) = B(J-1,2)                        INT14240
C     B(J,2) = 1.0 + EDVI                                               INT14250
40    CONTINUE                                                          INT14260
      JM     = 2                                                        INT14270
      BM     = B(2,2)                                                   INT14280
      DO 50 J=2,NP                                                      INT14290
      IF(BM.GT.B(J,2)) GOTO 50                                          INT14300
      JM     = J                                                        INT14310
      BM     = B(J,2)                                                   INT14320
50    CONTINUE                                                          INT14330
      GOTO 90                                                           INT14340
70    DO 80 J=JJ,NP                                                     INT14350
80    B(J,2) = 1.0 + EDVO*FINT(J)                                       INT14360
C  80 B(J,2) = 1.0 +EDVO                                                INT14370
C                                                                       INT14380
90    CONTINUE                                                          INT14390
      B(1,2) = 1.0                                                      INT14400
C                                                                       INT14410
      JJ     = 1                                                        INT14420
      DO 100 J=2,NP                                                     INT14430
```

```
      IF(U(J,2) .LT. 0.0) JJ = J
100   CONTINUE
      IF(JJ.EQ.1) GO TO 110
C
C   IN THE SEPARATED REGION, EDDY VISCOSITY IS SET EQUAL TO
C   THAT IN THE PREVOUS STATION TO AVOID NUMERICAL TROUBLE
      JJP3    = JJ + 3
      JJP3    = MIN(JJP3, NP)
      CALL AMEAN(1,JJP3,ETA,B(1,2),2)
110   CALL AMEAN(1,NP,ETA,B(1,2),1)
C
      RETURN
      END
C          DATA SET KCBCEXTRAP AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCEXTRAP AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCEXTRAP AT LEVEL 008 AS OF 02/13/84
      SUBROUTINE EXTRAP(NX,NXTE,X,Y)
C
C   EXTRAPOLATE DATA USING PARABOLIC FORMULA
      DIMENSION X(101),Y(101)
C - - - - - - - - - - - - - - - - - - - - - - - - -
      Y1     = Y(NX-2)
      Y2     = Y(NX-1)
      X1     = X(NX-2)
      X2     = X(NX-1)
      X3     = X(NXTE)
      X1     = X1 -X3
      X2     = X2 -X3
      BB     = (Y1-Y2)/(X1**2 - X2**2)
      AA     = Y1 - BB * X1**2
      DO 10 I=NX,NXTE
      X1     = X(I) -X3
      Y(I)   =  AA + BB * X1**2
10    CONTINUE
      RETURN
      END
C          DATA SET KCBCFILLUP AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCFILLUP AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCFILLUP AT LEVEL 007 AS OF 04/05/84
      SUBROUTINE FILLUP(INDEX)
      COMMON /BLC0/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON /BLC1/ F(101,2),U(101,2),V(101,2),W(101,2),B(101,2)
      COMMON /GRD / ETA(101),DETA(101),A(101)
C
C   -------------------------------------------------------
C
      IF(NP.GE.NPT ) RETURN
      IF(INDEX.EQ.2) GOTO 10
C
C   DEFINE PROFILES FOR B. L. GROWTH
      NP1     = NP + 1
      NP      = NP + 2
      NP      = MIN(NP, NPT)
      NM      = NP
      GOTO 20
C
```

```
C     FILL UP PROFILES BEFORE MOVING TO THE NEXT STATION
C
10     NP1     = NP + 1
       NM      = NPT
20     DO 30 J=NP1,NM
       F(J,2) = F(J-1,2) + DETA(J-1)*U(J-1,2)
       U(J,2) = U(J-1,2)
       V(J,2) = 0.0
       B(J,2) = B(J-1,2)
       W(J,2) = W(J-1,2)
30     CONTINUE
C
       RETURN
       END
C           DATA SET KCBCHEADER AT LEVEL 001 AS OF 08/24/84
C           DATA SET KCBCHEADER AT LEVEL 001 AS OF 08/24/84
C           DATA SET KCBCHEADER AT LEVEL 001 AS OF 04/05/84
       SUBROUTINE HEADER ( TITLE,SURFID,ISTRP )
       COMMON /ISURF/ ISF
C
       DIMENSION     TITLE(20),  SURFID(4)
C
10     FORMAT ( 1H1,20X,20A4 )
20     FORMAT ( 1H0,15X,'BOUNDARY LAYER CALCULATION FOR ',
     +          'UPPER SURFACE ',10X,'ICYCLE=',I5 / 16X,71(1H-) / )
30     FORMAT ( 1H0,15X,'BOUNDARY LAYER CALCULATION FOR ',
     +          'LOWER SURFACE ',10X,'ICYCLE=',I5 / 16X,71(1H-) / )
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
       WRITE ( 6,10 ) TITLE
       IF(ISF .EQ. 1) WRITE ( 6,20 ) ISTRP
       IF(ISF .EQ. 2) WRITE ( 6,30 ) ISTRP
C
       RETURN
       END
C           DATA SET KCBCINPUT  AT LEVEL 001 AS OF 08/24/84
C           DATA SET KCBCINPUT  AT LEVEL 001 AS OF 08/24/84
C           DATA SET KCBCINPUT  AT LEVEL 009 AS OF 08/24/84
       SUBROUTINE INPUT(ITR,ISWPT,SURFID)
       COMMON /BLC0/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
       COMMON /BLC1/ F(101,2),U(101,2),V(101,2),W(101,2),B(101,2)
       COMMON /BLC2/ DELF(101),DELU(101),DELV(101),DELW(101)
       COMMON /BLC7/ C(100,100),D(100),DB(100),DBP(100),UE0(100),GI
       COMMON /BONV/ ITMAX,EPSL,EPST,CONV
       COMMON/EDDY1/ RL,RX,SQRX,RXNTR,GMTR,GMTRS(100),
     +                   ALFAS(100),FFS(100),RTS(100),IEDY,NXSPT
       COMMON /GTY / X(101),UE(100),P1(100),P2(100),CEL,CELH
       COMMON /BLIN/ TITLE(20),XC(100),YC(100),ISG(100),DELS(100),
     +             XCTR,XTR,ISTRP,ICYCLE,ICYTL,XCTRS(2),TRFIND(2)
       COMMON/TRN/ PGAMTR,OMEGA,RTHETB,RTRANB
       COMMON /ISURF/ ISF
       DIMENSION D1(100),D2(100),D3(100)
       DIMENSION SURFID(4),XCS(100)
       LOGICAL TRFIND
C
```

```
C     --------------------------------------------------------------
C
      ITMAX    = 15
      EPSL     = 0.0001
      EPST     = 0.01
      NPT      = 101
      ETAE     = 8.0
      VGP      = 1.14
      DETA1    = 0.01
      NSS      = NXT / 4
C  SEARCH FOR PRESSURE PEAK AS SWITCH POINT
      UMAX = UE(1)
      DO 50 I = 2 , NXT
      IF (UE(I) .LE. UMAX) GO TO 55
      UMAX = UE(I)
50    CONTINUE
      GO TO 60
55    NS = I - 4
60    IF (NS .GT. NSS) NS = NSS
      IF (NS .LT.   3) NS = 3
C
C   CALCULATE THE PRESSURE PARAMETERS P1 + P2 FOR B. L. CALCULATION
C   USING TRANSFORMED COORDINATES
C
      CALL DIFF3 (NXT, X, UE, D1, D2, D3, 0 )
      DO 65 I = 2,NXT
      P2(I)   = X(I) * D1(I) /UE(I)
      P1(I)   = 0.5 * (1.0 + P2(I))
65    CONTINUE
      P1(1)   = 0.5 * (1.0 + P2(1))
      XCMIN   = XC(1)
      MIN     = 1
      DO 70 I=1,NXT
      IF(XCMIN.LT.XC(I)) GOTO 70
      XCMIN   = XC(I)
      MIN     = I
70    CONTINUE
      DO 80 I = 1 , NXT
      IF (I .GE. MIN) THEN
         XCS(I) = XC(I)
         ISG(I) = 1
      ELSE
         XCS(I) = -XCS(I)
         ISG(I) = -1
      END IF
80    CONTINUE
      INVRS   = NS + 1
C
C   SEARCH FOR TRANSITION LOCATION
      ITRP1   = ITR + 1
      GOTO (150, 95, 120, 150), ITRP1
C
C   TRANSITON LOCATION IS INPUT ( = XCTR)
 95   DO 100 I=1,NXT
      IF(XCTR.LT.XCS(I)) GOTO 105
100   CONTINUE
```

```
      NTR      = NXT + 1
      GOTO 200
105   NTR = I-1
      IF (NTR.LT.3) THEN
         NTR = 3
         XCTR = XC(NTR)
      END IF
      GOTO 200
C
C   TRANSITION LOCATION IS SET AT THE PRESSURE PEAK
C
120   UM       = UE(1)
      IM       = 1
      DO 75 I = 1,NXT
      IF(UM.GT.UE(I)) GOTO 75
      IM       = I
      UM       = UE(IM)
75    CONTINUE
      IF(IM.LT.4) IM = 4
      NTR   = IM
      XCTR = XC(NTR)
      GOTO 200
C
C   TRANSTION LOCATION WILL BE CALCULATED BASED ON MICHEL CRITERION
C
150   NTR      = NXT + 1
200   DO 210 I=1,NXT
210   GMTRS(I)= 0.0
C
C  TRANSITION LOCATION PROVISIONALLY FROM PREVIOUS CYCLE
C
      IF (TRFIND(ISF) ) THEN
         DO 211 I = 1 , NXT
         XCS(I) = XC(I)
         IF (I .LT. MIN) XCS(I) = -XCS(I)
211      CONTINUE
         DO 215 I=1,NXT
         IF(XCTRS(ISF) .LE.XCS(I)) GOTO 217
215      CONTINUE
217      NTR = I-1
         XCTR = XCTRS(ISF)
         IF (NTR .LT. 3) THEN
              NTR = 3
              XCTR = XC(NTR)
         END IF
      END IF
C
C  CALCULATE GAMTR DISTRIBUTION
C
      IF (NTR.GT.NXT-1) GOTO 250
      FAC  = (XCTR-XC(NTR))/(XC(NTR+1)-XC(NTR))
      XTR  = X(NTR) + FAC*(X(NTR+1)-X(NTR))
      UETR = UE(NTR) + FAC*(UE(NTR+1)-UE(NTR))
      RXNTR  = XTR * UETR * RL
      GGFT   = 1.0/PGAMTR*RL**2/RXNTR**1.34
      DO 220 I=NTR+1,NXT
```

```
      ALFAS(I) = 0.0168
220   GMTRS(I)= 1.0
      ALFAS(NTR) = 0.0168
      UEINTG = 0.0
      U1 = 0.5/UETR
      X1 = XTR
      DO 230 I=NTR+1,NXT
      U2 = 0.5/UE(I)
      X2 = X(I)
      UEINTG = UEINTG+(U1+U2)*(X2-X1)
      U1 = U2
      X1 = X2
      GG = GGFT*UEINTG*(X(I)-XTR)*UE(I)**3
      IF(GG .GT. 10.0) GO TO 250
      GMTRS(I) = 1.0-EXP(-GG)
230   CONTINUE
250   CONTINUE
C
C   GENERATE B. L. ETA GRIDS + INTIAL VELOCITY PROFILES
C
      CALL INTL(ETAE,DETA1,VGP)
      DO 260 I=1,NXT
      UEO(I)  = UE(I)
260   CONTINUE
C   PRINT OUT INPUT DATA
C
      IF (ICYCLE .GE. ICYTL-1 .OR. IP .GE. 0) THEN
         CALL HEADER( TITLE,SURFID,ISTRP )
         WRITE(6,1002) NXT,ITR,IP,NS,NTR,ISWPT
         WRITE(6,1003) VGP,DETA1,RL,XCTR,OMEGA,PGAMTR
      ELSE
         IF (ISF.EQ.1) WRITE(6,1008) ICYCLE
         IF (ISF.EQ.2) WRITE(6,1009) ICYCLE
      END IF
      IF (NTR.LT.NXT) THEN
         IF (ITR.EQ.1) WRITE (6,1005) XCTR,XTR,NTR
         IF (ITR.EQ.2) WRITE (6,1006) XCTR,XTR,NTR
         IF (TRFIND(ISF)) WRITE(6,1007) XCTR,XTR,NTR
      END IF
      RETURN
C
C   --------------------------------------------------------------
C
2     FORMAT(20A4)
3     FORMAT(10I5)
4     FORMAT(6F10.0)
1001  FORMAT(1H1,20X,20A4)
1002  FORMAT(1H0,10H   NXT   = ,I5,7X,10H   ITR   = ,I5,7X/
     +            1H ,10H   IP    = ,I5,7X,10H   NS    = ,I5,7X/
     +            1H ,10H   NTR   = ,I5,7X,10H   ISWPT= ,I5)
1003  FORMAT(1H0,10H   VGP   = ,E12.4,10H   DETA1= ,E12.4/
     +            1H ,10H   RL    = ,E12.4,10H   XCTR = ,E12.4/
     +            1H ,10H  OMEGA = ,E12.4,10H  PGAMTR= ,E12.4)
1004  FORMAT(1H0,3X,2H I,6X,2HXC,11X,2HYC,11X,2H X,11X,2HUE,11X,2HP1,
     +            11X,2HP2,/(1H ,3X,I3,6E13.5))
1005  FORMAT(/3X,'BEGIN OF TRANSITION IS BEING INPUT AT X/C =',F8.4,4X,
```

```
     +        'S/C =',F8.4,4X,'NTR =',I3/)
 1006 FORMAT(/3X,'BEGIN OF TRANSITION IS SET AT PRESSURE PEAK, X/C =',
     +        F8.4,4X,'S/C =',F8.4,4X,'NTR =',I3/)
 1007 FORMAT(/3X,'BEGIN OF TRANSITION IS PROVISIONALLY TAKEN FROM ',
     +  'PREVIOUS CYCLE: X/C=',F8.4,4X,'S/C =',F8.4,4X,'NTR =',I3/)
 1008 FORMAT(//3X,'UPPER SURFACE CALCULATIONS OF CYCLE',I3)
 1009 FORMAT(//3X,'LOWER SURFACE CALCULATIONS OF CYCLE',I3)
      END
C          DATA SET KCBCINTL   AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCINTL   AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCINTL   AT LEVEL 009 AS OF 02/22/84
      SUBROUTINE INTL(ETAE,DETA1,VGP)
      COMMON /BLC0/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON /BLC1/ F(101,2),U(101,2),V(101,2),W(101,2),B(101,2)
      COMMON /BLC2/ DELF(101),DELU(101),DELV(101),DELW(101)
      COMMON /BLC6/ S1(101),S2(101),S3(101),S4(101),S5(101),S6(101),
     +                    S7(101),S8(101),R1(101),R2(101),R3(101),R4(101)
      COMMON /BCNV/ ITMAX,EPSL,EPST,CONV
      COMMON /GRD / ETA(101),DETA(101),A(101)
      COMMON /GTY / X(101),UE(100),P1(100),P2(100),CEL,CELH
C
C     -----------------------------------------------------------------
C
C    GENERATE THE GRID
C
      DETA(1) = DETA1
      IF(VGP.LT.1.0) VGP = 1.0
      IF((VGP-1.0) .LE. 0.001)  GO TO 10
      NP     = ALOG((ETAE/DETA(1))*(VGP-1.0)+1.0)/ALOG(VGP) + 1.001
      GO TO 20
 10   NP     = ETAE/DETA(1) + 1.001
 20   IF(NP .LE. NPT)  GO TO 30
      WRITE(6, 150 )
      STOP
 30   ETA(1) = 0.0
      DO 40 J=2,NPT
      ETA(J) = ETA(J-1) + DETA(J-1)
      DETA(J)= VGP*DETA(J-1)
      A(J)   = 0.5*DETA(J-1)
 40   CONTINUE
C
C   GENERATE INITIAL VELOCITY PROFILE
 80   DO 90  J=1,NP
      ETAB = ETA(J)/ETA(NP)
      ETAB2 = ETAB**2
      F(J,2) = 0.25*ETA(NP)*ETAB2*(3.0 - 0.5*ETAB2)
      U(J,2) = 0.5*ETAB*(3.0 - ETAB2)
      V(J,2) = 1.5*(1.0 - ETAB2)/ETA(NP)
      B(J,2) = 1.0
      W(J,2) = 1.0
 90   CONTINUE
      NX     = 1
      IT     = 0
 120  IT     = IT + 1
      IF(IT .LT. ITMAX) GO TO 130
```

```
      STOP
130   CONTINUE
C
      DO 140 J= 2,NP
      FB     = 0.5*(F(J,2) + F(J-1,2))
      UB     = 0.5*(U(J,2) + U(J-1,2))
      VB     = 0.5*(V(J,2) + V(J-1,2))
      USB    = 0.5*(U(J,2)**2 + U(J-1,2)**2)
      DERBV  = (B(J,2)*V(J,2) - B(J-1,2)*V(J-1,2))/DETA(J-1)
      FVB    = 0.5*(F(J,2)*V(J,2) + F(J-1,2)*V(J-1,2))
C
      S1(J)  =  0.5*P1(NX)*F(J,2) + B(J,2)/DETA(J-1)
      S2(J)  =  0.5*P1(NX)*F(J-1,2) - B(J-1,2)/DETA(J-1)
      S3(J)  = 0.5*P1(NX)*V(J,2)
      S4(J)  = 0.5*P1(NX)*V(J-1,2)
      S5(J)  = -P2(NX)*U(J,2)
      S6(J)  = -P2(NX)*U(J-1,2)
      CRB    = -P2(NX)
      R2(J)  = CRB - (DERBV + P1(NX)*FVB - P2(NX)*USB)
C
      R1(J)  = F(J-1,2) - F(J,2) + DETA(J-1)*UB
      R3(J-1)= U(J-1,2) - U(J,2) + DETA(J-1)*VB
140   CONTINUE
      R1(1)  = 0.0
      R2(1)  = 0.0
      R3(NP) = 0.0
      CALL SOLV3
      IF(ABS(DELV(1)) .GT. EPSL  ) GO TO 120
      CALL FILLUP(2)
      CALL OUTPUT(1)
C
      RETURN
C
150   FORMAT(1H0,37HNP EXCEEDED NPT -- PROGRAM TERMINATED)
      END
C          DATA SET KCBCINTRP3 AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCINTRP3 AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCINTRP3 AT LEVEL 003 AS OF 04/05/84
      SUBROUTINE INTRP3 (N1,X1,F1,FP1,FPP1,FPPP1,N2,X2,F2)
C
C     CUBIC INTERPOLATION
C
C     GIVEN THE VALUES OF A FUNCTION (F1) AND ITS DERIVATIVES
C     AT N1 VALUES OF THE INDEPENDENT VARIABLE (X1)
C
C     FIND THE VALUES OF THE FUNCTION (F2)
C     AT N2 VALUES OF THE INDEPENDENT VARIABLE (X2)
C
C     X2 CAN BE IN ARBITRARY ORDER
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
      DIMENSION X1(101),F1(101),FP1(101),FPP1(101),FPPP1(101),X2(101),
     +             F2(101)
      DATA EPS /1E-04/
C
```

```
      JT = 2
      DO 10 I = 1,N2
      XM      = X2(I)
      DO 20 J = JT,N1
      J1 = J -1
      IF (X1(J).GE.XM ) GO TO 30
20    CONTINUE
      J = N1
30    JT = J
      DXX     = X2(I) - X1(J1)
      DXX2    = DXX  * DXX / 2.
      DXX3    = DXX2 * DXX / 3.
10    F2(I)   = F1(J1) + DXX*FP1(J1) + DXX2*FPP1(J1) + DXX3*FPPP1(J1)
C
      RETURN
      END
C          DATA SET KCBCJOIN   AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCJOIN   AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCJOIN   AT LEVEL 012 AS OF 02/20/84
      SUBROUTINE JOIN(INDEX)
      COMMON /BLC0/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON /BLC1/ F(101,2),U(101,2),V(101,2),W(101,2),B(101,2)
      COMMON /BLC7/ C(100,100),D(100),DB(100),DBP(100),UE0(100),GI
      COMMON/EDDY1/ RL,RX,SQRX,RXNTR,GMTR,GMTRS(100)
     +               ,ALFAS(100),FFS(100),RTS(100),IEDY,NXSPT
      COMMON /GTY / X(101),UE(100),P1(100),P2(100),CEL,CELH
      COMMON /GRD / ETA(101),DETA(101),A(101)
      COMMON /SAVE/ FS(101),US(101),VS(101),BS(101),WS(101)
      COMMON /SMRY/ VW(100),ITP(100),ISL(100),DLS(100),CF(100),
     +              THT(100),NPSTR(100)
C
C    --------------------------------------------------------------
C
C   INDEX = 1 FOR THE FIRST SWEEP
C   INDEX = 2 FOR SUBSEQUENT SWEEP
C
      CALL COMPGI
      CII    = C(NX,NX)
      UES    = GI / (1.0 - DLS(NX) * SQRT(RL) * CII )
      IF(INDEX.EQ.1) GOTO 15
C
C    RETRIEVE PROFILES AT STATION NS FOR INVERSE B. L.
C    CALCULATION
      DO 10 J=1,NPT
      F(J,2) = FS(J)
      U(J,2) = US(J)
      V(J,2) = VS(J)
      W(J,2) = WS(J)
      B(J,2) = BS(J)
10    CONTINUE
      UES    = UES/W(1,2)
      SQS    = 1.0
      GOTO 30
15    CONTINUE
      SQS    = 1.0 / SQRT(UES)
      DO 20 J=2,NPT
```

```
      ETA(J) = ETA(J)*SQS
      DETA(J-1) = ETA(J) - ETA(J-1)
      A(J)   = 0.5*DETA(J-1)
20    CONTINUE
C
30    DO 60 J=1,NPT
      U(J,2) = U(J,2)*UES
      W(J,2) = UES * W(J,2)
      F(J,2) = F(J,2)*SQS*UES
      V(J,2) = V(J,2)/SQS*UES
60    CONTINUE
      UE(NX) = W(1,2)
      RX     = UE(NX)*X(NX)*RL
      SQRX   = SQRT(RX)
C     IF(NX.GT.NTR) CALL EDDY
      CALL FILLUP(2)
      IF(INDEX.EQ.2) GOTO 70
C
C   STORE PROFILES AT STATION NS FOR THE NEXT SWEEP
      DO 65 J=1,NPT
      FS(J)  = F(J,2)
      US(J)  = U(J,2)
      VS(J)  = V(J,2)
      WS(J)  = W(J,2)
      BS(J)  = B(J,2)
65    CONTINUE
70    DO 80 J=1,NPT
      F(J,1) = F(J,2)
      U(J,1) = U(J,2)
      V(J,1) = V(J,2)
      W(J,1) = W(J,2)
      B(J,1) = B(J,2)
80    CONTINUE
      RETURN
      END
C          DATA SET KCBCMAIN   AT LEVEL 005 AS OF 09/18/84
C
C      PROGRAM MAIN
C
C--------------------------------------------------
      SUBROUTINE CASBLP(K2,XP,YP,XMP,YMP,XS,YS,DLSP,VC,DBPP
     +             ,RN,NBL,ITRI,XCTRI,CASEID)
      COMMON /BLCO/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON/BLIN/ TITLE(20),XC(100),YC(100),ISG(100),DELS(100),
     +           XCTR,XTR,ISTRP,ICYCLE,ICYTL,XCTRS(2),TRFIND(2)
      COMMON/EDDY1/RL,RX,SQRX,RXNTR,GMTR,GMTRS(100),
     +             ALFAS(100),FFS(100),RTS(100),IEDY,NXSPT
      COMMON/BLOW/ VN(100)
      COMMON/TRN/ PGAMTR,OMEGA,RTHETB,RTRANB
      COMMON/PLOT/NVP(2),NXVP(20,2),ICC
      DIMENSION     CASEID( 20 ),  XCTRI (  2 ),  ITRI  (  2 )
      DIMENSION     XP    ( 100),  YP    ( 100),  XMP   ( 100)
      DIMENSION     YMP   ( 100),  VC    ( 100),  SM    ( 100)
      DIMENSION     XS    ( 100),  YS    ( 100),  NBL   ( 2 )
      DIMENSION     VT    ( 100),  S     ( 100),  DLSP  ( 100)
      DIMENSION     DLS   ( 100),  XO    ( 100),  YO    ( 100)
```

```
      DIMENSION    D1    ( 100),  D2    ( 100),  D3    ( 100)
      DIMENSION    XPS   (100),  YPS(100) , DBPP(100)
C
10    FORMAT ( 20A4 )
20    FORMAT ( 4I5 )
30    FORMAT ( 3I5,3F10.0 )
40    FORMAT ( 6F10.5 )
100   FORMAT ( 1H1,5X,20A4 )
110   FORMAT ( 1H0,6X,'CYCLE NO. = ',I5 )
120   FORMAT ( 1H0,6X,'FINISHED TOTAL NUMBER OF CYCLES = ',I5,
     +             4X,'JOB COMPLETED.' )
130   FORMAT ( 1H0,6X,'THE UPDATED DISPLACEMENT SURFACE',/1X,2HNX,
     +             9X,2HXP,12X,2HYP,11X,3HDLS,10X,4HDBPP,12X,2HVN)
140   FORMAT ( I5,5E14.6)
150   FORMAT ( 1H0.6X,'READ IN CONTROL POINTS DATA',/1X,2HNX,9X,
     +                 3HXMP,11X,3HYMP,12X,2HSM,13X,2HVC)
160   FORMAT ( I5,4E14.6)
170   FORMAT ( 1H0,6X,'INTERPOLATED ORIGINAL GEOMETRY DATA',/1X,
     +                 2HNX,9X,2HXP,12X,2HYP,12X,1HS,14X,2HVT)
180   FORMAT (I5,4E14.6)
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
      GRANG(X1,X2,X3,Y1,Y2,Y3,X0)= (X0-X2)*(X0-X3)/(X1-X2)/(X1-X3)*Y1
     +          +(X0-X1)*(X0-X3)/(X2-X1)/(X2-X3)*Y2+(X0-X1)*(X0-X2)
     +          /(X3-X1)/(X3-X2)*Y3
      ISWPT = 1
      IEDY  = 1
      NBL(1) = 91
      NBL(2) = 71
C
C     WRITE ( 6,100 ) CASEID
5     CONTINUE
      ISTRP = ICYCLE
      NN = K2 - 1
      NT = K2
C      WRITE ( 6,110 ) ICYCLE
C  INTERPOLATE OUTPUT CONTROL POINTS TO ORIGINAL GEOMETRY
C
      DO 15 I = 1 , NT
      XPS(I) = XP(I)
      YPS(I) = YP(I)
15    CONTINUE
      S(1) = 0.0
      DO 50 I = 2 , NT
      S(I) = S(I-1) + SQRT((XP(I)-XP(I-1))**2 +
     +                          (YP(I)-YP(I-1))**2)
50    CONTINUE
      SM(1) = SQRT((XMP(1)-XP(1))**2 + (YMP(1)-YP(1))**2)
      DO 60 I = 2 , NN
      SM(I) = SM(I-1) + SQRT((XMP(I)-XMP(I-1))**2+
     +                          (YMP(I)-YMP(I-1))**2)
60    CONTINUE
C     CALL AMEAN(NN-10,NN,SM,VC,1)
      CALL AMEAN(1,NN,SM,VC,1)
      SNT = S(NT)
```

```
      SM(NT) = SM(NN)+SQRT((XMP(NN)-XP(NT))**2+(YMP(NN)-YP(NT))**2)
      SMNT = S(NT) / SM(NT)
      DO 65 I = 1 , NN
65    SM(I) = SM(I) * SMNT
      CALL DIFF3(NN,SM,VC,D1,D2,D3,0)
      CALL INTRP3(NN,SM,VC,D1,D2,D3,NT,S,VT)
C  PRINT OUT INPUT DATA
C
C     WRITE(6,150)
C     WRITE(6,160) (I,XMP(I),YMP(I),SM(I),VC(I),I=1,NN)
C     WRITE(6,170)
C     WRITE(6,180) (I,XP(I),YP(I),S(I),VT(I),I=1,NT)
C
      XPMIN = XP(1)
      DO 44 I = 2 , NT
      IF (XP(I) .GT. XPMIN) GO TO 44
      XPMIN = XP(I)
44    CONTINUE
      CHORD = XP(NT) - XPMIN
      DO 45 I = 1 , NT
      XP(I) = (XP(I)-XPMIN) / CHORD
      YP(I) = YP(I) / CHORD
45    CONTINUE
      CALL COMPBL ( CASEID,XP,YP,VT,S,DLSP,DLS,DBPP,NBL,ITRI,XCTRI,
     +                  RN,NT,ISWPT)
C
C
C  SMOOTH THE CALCULATED DISPLACEMENT THINKNESS
C
C     CALL SMFIT(1,NT,S,DLS,D1,2)
C
C  ADD DISPLACEMENT THINKNESS ON THE ORIGINAL BODY
C
      DO 70 I = 1 , NT
      DLSP(I) = DLS(I)
70    CONTINUE
C     CALL SMFIT (1,NT,XS,YS,D1,2)
      DO 80 I = 1 , NT
      XP(I) = XPS(I)
      YP(I) = YPS(I)
80    CONTINUE
      IF (ICYCLE .GE. ICYTL-1 .OR. IP .GE. 0) THEN
C      WRITE (6,130)
C      WRITE (6,140) (I,XP(I),YP(I),DLS(I),DBPP(I),VN(I),I=1,NT)
      WRITE ( 6,120 ) ICYCLE
      END IF
C
      RETURN
      END
C          DATA SET KCBCMAIN2  AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCMAIN2  AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCMAIN2  AT LEVEL 010 AS OF 04/06/84
      SUBROUTINE MAIN2(ITR,ISWPT,SURFID)
      COMMON /BLC0/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON /BLC1/ F(101,2),U(101,2),V(101,2),W(101,2),B(101,2)
      COMMON /BLC2/ DELF(101),DELU(101),DELV(101),DELW(101)
```

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      COMMON /BLC7/ C(100,100),D(100),DB(100),DBP(100),UE0(100),GI      INT20590
      COMMON /BONV/ ITMAX,EPSL,EPST,CONV                                INT20600
      COMMON/EDDY1/ RL,RX,SQRX,RXNTR,GMTR,GMTRS(100)                    INT20610
     +                    ,ALFAS(100),FFS(100),RTS(100),IEDY,NXSPT      INT20620
      COMMON /GTY / X(101),UE(100),P1(100),P2(100),CEL,CELH             INT20630
      COMMON /SMRY/ VW(100),ITP(100),ISL(100),DLS(100),CF(100),         INT20640
     +                     THT(100),NPSTR(100)                          INT20650
      COMMON /BLIN/ TITLE(20),XC(100),YC(100),ISG(100),DELS(100),       INT20660
     +              XCTR,XTR,ISTRP,ICYCLE,ICYTL,XCTRS(2),TRFIND(2)      INT20670
      COMMON/BLC9/ UEB(100) , CFS(100)                                  INT20680
      COMMON/TRN/ PGAMTR,OMEGA,RTHETB,RTRANB                            INT20690
      COMMON /ISURF/ ISF                                                INT20700
      COMMON/PLOT/NVP(2),NXVP(20,2),ICC                                 INT20710
      DIMENSION SURFID(4),RTSS(11)                                      INT20720
      LOGICAL SMOOTH , SEPART , HOMOPY                                  INT20730
      DATA RTSS/0.0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0/            INT20740
                                                                        INT20750
C   --------------------------------------------------------------------INT20760
C                                                                       INT20770
      GRANG(X1,X2,X3,Y1,Y2,Y3,X0)= (X0-X2)*(X0-X3)/(X1-X2)/(X1-X3)*Y1   INT20780
     +          +(X0-X1)*(X0-X3)/(X2-X1)/(X2-X3)*Y2+(X0-X1)*(X0-X2)     INT20790
     +          /(X3-X1)/(X3-X2)*Y3                                     INT20800
      ISWP   = 0                                                        INT20810
      INDEX  = 1                                                        INT20820
      IGROWT = 2                                                        INT20830
      NXSPT = NXT + 1                                                   INT20840
10    CALL JOIN(INDEX)                                                  INT20850
      NXSTOP = NXT-1                                                    INT20860
      IF (NS .GE. NTR ) GOTO 15                                         INT20870
15    ISWP   = ISWP + 1                                                 INT20880
20    NX     = NX + 1                                                   INT20890
      HOMOPY = .FALSE.                                                  INT20900
25    CEL    = 0.5*(X(NX)+X(NX-1))/(X(NX)-X(NX-1))                      INT20910
      P1(NX) = 0.5                                                      INT20920
      P2(NX) = 0.0                                                      INT20930
      CELH   = 0.5*CEL                                                  INT20940
30    IT     = 0                                                        INT20950
      CALL COMPGI                                                       INT20960
      IGROW=1                                                           INT20970
70    IT     = IT + 1                                                   INT20980
      RX     = UE(NX)*X(NX)*RL                                          INT20990
      SQRX   = SQRT(RX)                                                 INT21000
C                                                                       INT21010
      IF(IT .LE. ITMAX) GO TO 80                                        INT21020
      IF(HOMOPY) GO TO 72                                               INT21030
      IRC = 1                                                           INT21040
      RT  = RTSS(IRC)                                                   INT21050
      HOMOPY = .TRUE.                                                   INT21060
      UEREF  = UE0(NX-1)                                                INT21070
      UESAVE = UE0(NX)                                                  INT21080
      UE0(NX) = RT*UESAVE+(1.0-RT)*UEREF                                INT21090
      DO 61 J=1,NP                                                      INT21100
      F(J,2) = F(J,1)                                                   INT21110
      U(J,2) = U(J,1)                                                   INT21120
      V(J,2) = V(J,1)                                                   INT21130
      W(J,2) = W(J,1)                                                   INT21140
```

```
      B(J,2) = B(J,1)
61    CONTINUE
      GO TO 30
C
72    NXSTOP = NX - 1
      CALL AMEAN(NS,NXSTOP,X,CF,1)
C     CALL AMEAN(NS,NXSTOP,X,VW,1)
C     CALL HEADER( TITLE,SURFID,ISTRP )
      WRITE (6, 250 ) ISWP
      WRITE (6, 260 ) (M,XC(M),X(M),CF(M),DLS(M),THT(M),UE(M),
     +    UE0(M),D(M),DB(M),GMTRS(M),ITP(M),NPSTR(M),M=1,NXSTOP)
      WRITE(6, 270 ) NX
      STOP
80    CONTINUE
      IF(NX .GT. NTR) GOTO 100
C
C   LAMINAR FLOW CALCULATION
C
      CALL COEF(GAMMA1,GAMMA2)
      CALL SOLV4(GAMMA1,GAMMA2)
      UE(NX) = U(NP,2)
      IF(ABS(DELV(1)) .GT. EPSL) GO TO 70
C
C  CHECK ON LAMINAR FLOW SEPARATION. IF SEPARATION OCCURS, ASSIGN BEGIN
C  OF TRANSITION TO THAT POINT AND RECOMPUTE THE CURRENT STATION NX
C
      IF(V(1,2).GT.0.0 .OR. ITR.NE.3) GOTO 110
      CALL TRNS(ICODE)
      GOTO 25
C
C   TURBULENT FLOW CALCULATION
C
100   CONTINUE
      CALL EDDY
      CALL COEF(GAMMA1,GAMMA2)
      CALL SOLV4(GAMMA1,GAMMA2)
      UE(NX) = U(NP,2)
      VM     = AMAX1(V(1,2),1.0)
      IF(ABS(DELV(1)/VM) .GT. EPST) GO TO 70
110   CONTINUE
C
C     CHECK FOR B. L. GROWTH
C
      IF(NP .GE. NPT) GO TO 120
      IF(ABS(V(NP,2)) .LT. 0.0005 .AND. ABS(1.0-U(NP-2,2)/U(NP,2))
     +                 .LT.0.0035.OR.IGROW.GT.IGROWT) GOTO 120
      CALL FILLUP(1)
      IGROW=IGROW+1
      IT     = 1
      GO TO 70
C
120   CONTINUE
      CALL FILLUP(2)
      CALL OUTPUT(2)
      IF(NX.GE.NTR  .OR.  ITR.EQ.0) GOTO 150
```

```
      IF(NX.LT.3 .OR.  ITR.NE.3) GOTO 150
C
C    CALCULATE TRANSITION LOCATION USING MICHEL METHOD
C
      CALL TRNS(ICODE)
      IF(ICODE.EQ.0) GOTO 150
C
C    TRANSITION OCCURS BASED ON MICHEL CRITERIOR AT STATION NX
C    RECALCULATE B. L. AT NX STATION ASSUMING THE FLOW IS TRANSITIONAL
C
      IT      = 0
      IGROW   = 1
      GOTO 70
150   CONTINUE
      IF(.NOT. HOMOPY ) GO TO 154
      IF( RT .GT. 0.9999) GO TO 154
      IRC = IRC + 1
      RT  = RTSS(IRC)
      UEO(NX) =RT*UESAVE + (1.0-RT)*UEREF
      GO TO 30
154   CONTINUE
      IF(NX .LT. NXSTOP) GO TO 20
C
C    THE B. L. CALCULATION FOR THE CURRENT SWEEP IS COMPLETED.
C    CHECK FOR THE CONVERGENCE AND , IT NOT, MOVE TO THE NEXT
C    SWEEP.
C
C
160   CONTINUE
      D(NXT)  = GRANG(X(NXT-3),X(NXT-2),X(NXT-1),D(NXT-3),D(NXT-2),
     +               D(NXT-1),X(NXT))
      DLS(NXT)= GRANG(X(NXT-3),X(NXT-2),X(NXT-1),DLS(NXT-3),DLS(NXT-2),
     +               DLS(NXT-1),X(NXT))
      UE(NXT) = GRANG(X(NXT-3),X(NXT-2),X(NXT-1),UE(NXT-3),
     +                    UE(NXT-2),UE(NXT-1),X(NXT))
      DO 165 I = 1 , NXSTOP
165   CFS(I) = CF(I)
      CALL AMEAN(NS,NXSTOP,X,CF,1)
C     CALL AMEAN(NS,NXSTOP,X,VW,1)
C     CALL HEADER( TITLE,SURFID,ISTRP )
      IF(ICYCLE .LT. ICYTL-1 .AND. IP .LT. 0)GO TO 170
      WRITE (6, 250 ) ISWP
      WRITE (6, 262 ) 1,XC(1),X(1),CF(1),DLS(1),THT(1),UE(1),
     +                UEO(1),0.0,GMTRS(1),ITP(1),NPSTR(1)
      WRITE (6, 264 ) (M,XC(M),X(M),CF(M),DLS(M),THT(M),UE(M),
     +    UEO(M),DLS(M)/THT(M),GMTRS(M),ITP(M),NPSTR(M),M=2,NXSTOP)
      IF ((ICYCLE.EQ.ICYTL).AND.(IP.EQ.-2).AND.(NVP(ISF).NE.0)) THEN
         WRITE(12,800) NS+1,NTR,XCTR
         WRITE(12,810) (XC(M),X(M),UE(M),CF(M),GMTRS(M),ISG(M),
     2                 M=1,NXSTOP)
 800     FORMAT(2I5,F10.6)
 810     FORMAT(5E15.5,I5)
      END IF
170   CONTINUE
C
      DMAX   = D(1)
```

```
      DDMAX  = ABS(D(1) - DB(1))                                        INT22260
      DO 180 I = 2,NXT                                                  INT22270
      DMAX   = AMAX1( DMAX,D(I) )                                       INT22280
      DD     = ABS(D(I) - DB(I))                                        INT22290
      DDMAX  = AMAX1( DDMAX,DD )                                        INT22300
  180 CONTINUE                                                          INT22310
      IF ( ABS( DDMAX / DMAX ) .LE. 0.0050 ) RETURN                     INT22320
C                                                                       INT22330
C                                                                       INT22340
C    UPDATE D FOR THE NEXT SWEEP                                        INT22350
C                                                                       INT22360
      IF (ISWP .GT. 1) GO TO 195                                        INT22370
      DO 190 I = NS , NXT                                               INT22380
  190 D(I) = D(I)*(1.0+OMEGA*(UE(I)/UEO(I)-1.0))                        INT22390
      GO TO 205                                                         INT22400
  195 IF (ISWP .EQ. 2) GOTO 205                                         INT22410
      DO 200 I = NS , NXT                                               INT22420
  200 D(I) = D(I) * (1.0+OMEGA*(UE(I)/UEB(I)-1.0))                      INT22430
  205 IF(ISWP .GE. ISWPT ) RETURN                                       INT22440
      NX      = NS                                                      INT22450
      NP      = NPSTR(NX)                                               INT22460
      INDEX   = 2                                                       INT22470
      DO 210 I= 1,NXT                                                   INT22480
      DB(I)   = D(I)                                                    INT22490
      UEB(I) = UE(I)                                                    INT22500
  210 CONTINUE                                                          INT22510
      GOTO 10                                                           INT22520
C --------------------------------------------------------------       INT22530
  250 FORMAT(1H0,' ** SUMMARY OF INVERSE BOUNDARY LAYER SOLUTIONS. **',/INT22540
     +            1H0,4X,'ISWP  =',I3/)                                 INT22550
  260 FORMAT(1H0,4X,2HNX,5X,3HX/C,9X,1HX,9X,2HCF, 8X,                   INT22560
     +            3HDLS,8X,3HTHT,9X,2HUE, 8X,3HUEO,10X,1HD ,9X,2HDB,3X, INT22570
     +       4HGMTR,4X,2HIT,1X,2HNP/(1H ,3X,I3,F10.5,8E11.4,F8.4,2I3))  INT22580
  262 FORMAT(1H0,4X,2HNX,6X,3HX/C,11X,1HX,10X,2HCF,9X,                  INT22590
     +            3HDLS,9X,3HTHT,10X,2HUE,9X,3HUEO,11X,1HH,8X,          INT22600
     +       4HGMTR,4X,2HIT,1X,2HNP/(1H ,3X,I3,9E12.4,2I3))             INT22610
  264 FORMAT(1H ,3X,I3,9E12.4,2I3)                                      INT22620
  270 FORMAT(1H0,' ** ITERATIONS EXCEEDED ITMAX AT NX =',I5,',**',/     INT22630
     +            1H0,' ** CALCULATIONS STOP. **')                      INT22640
      END                                                               INT22650
C         DATA SET KCBCOUTPUT AT LEVEL 001 AS OF 08/24/84               INT22660
C         DATA SET KCBCOUTPUT AT LEVEL 001 AS OF 08/24/84               INT22670
C         DATA SET KCBCOUTPUT AT LEVEL 002 AS OF 02/22/84               INT22680
      SUBROUTINE OUTPUT(INDEX)                                          INT22690
      COMMON /BLCO/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP                    INT22700
      COMMON/BLIN/ TITLE(20),XC(100),YC(100),ISG(100),DELS(100),        INT22710
     +             XCTR,XTR,ISTRP,ICYCLE,ICYTL,XCTRS(2),TRFIND(2)       INT22720
      COMMON /BLC1/ F(101,2),U(101,2),V(101,2),W(101,2),B(101,2)        INT22730
      COMMON /BLC7/ C(100,100),D(100),DB(100),DBP(100),UEO(100),GI      INT22740
      COMMON/EDDY1/ RL,RX,SQRX,RXNTR,GMTR,GMTRS(100)                    INT22750
     +                   ,ALFAS(100),FFS(100),RTS(100),IEDY,NXSPT       INT22760
      COMMON /GTY / X(101),UE(100),P1(100),P2(100),CEL,CELH             INT22770
      COMMON /GRD / ETA(101),DETA(101),A(101)                           INT22780
      COMMON /SMRY/ VW(100),ITP(100),ISL(100),DLS(100),CF(100),THT(100),INT22790
     +                   NPSTR(100)                                     INT22800
      COMMON /ISURF/ ISF                                                INT22810
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```
      COMMON/PLOT/NVP(2),NXVP(20,2),ICC
C
C   -----------------------------------------------------------------
C
      ITP(NX) = IT
      NPSTR(NX)=NP
      IF(NX.GT.1) GOTO 5
      DLS(NX)= 0.0
      VW(NX) = 0.0
      D(NX)  = 0.0
      THT(NX)= 0.0
      CF(NX) = 0.0
      VW(NX) = 0.0
      GOTO 150
5     GOTO (10,100,200), INDEX
C
C   CALCULATE B. L. PARAMETERS FOR TRANSFORMED COORDINATES
10    CONTINUE
      CF(NX) = 2.0 * V(1,2) * B(1,2)/SQRX
      VW(NX) = UE(NX)*SQRT(UE(NX)/X(NX))*V(1,2)
      DLS(NX)= X(NX)/SQRX * (ETA(NP)-F(NP,2))
      D(NX)  = UE(NX) * DLS(NX) * SQRT(RL)
      U1     = U(1,2) * (1.0 -U(1,2))
      SUM    = 0.0
      DO 20 J=2,NP
      U2     = U(J,2) * (1.0 -U(J,2))
      SUM    = SUM + A(J) * (U1 + U2)
      U1     = U2
20    CONTINUE
      THT(NX)= X(NX)/SQRX * SUM
      GOTO 150
C
C   CALCULATE B. L. PARAMETERS FOR SEMI-TRANSF COORDINATES
100   CONTINUE
      SQXC = SQRT(X(NX))
      SQRL = SQRT(RL)
      CF(NX) = 2.0 * V(1,2) * B(1,2)/(SQXC*SQRL*W(NP,2)**2)
      VW(NX) = V(1,2) / SQXC
      UE(NX) = U(NP,2)
      RX = RL * UE(NX) * X(NX)
      DLS(NX) = (ETA(NP)-F(NP,2)/U(NP,2))/SQRL*SQXC
      SUM     = 0.0
      U1      = U(1,2)/U(NP,2)*(1.0 -U(1,2)/U(NP,2))
      DO 120 J=2,NP
      U2      = U(J,2)/U(NP,2)*(1.0 -U(J,2)/U(NP,2))
      SUM     = SUM + A(J) * (U1 + U2)
      U1      = U2
120   CONTINUE
      THT(NX) = SUM /SQRL * SQXC
      D(NX)   = (U(NP,2)*ETA(NP)-F(NP,2)) * SQXC
150   IF (NX .GE. NXT) GO TO 160
      IF (IEDY .EQ. 0 .OR. NX .LE. NTR+2) GO TO 160
C
C  MODIFY ALFA USING SIMPSON'S ARGUMENTS
C
```

```
      CALL SMPSON
160   DO 175 J=1,NPT
      F(J,1)  = F(J,2)
      U(J,1)  = U(J 2)
      V(J,1)  = V(J,2)
      W(J,1)  = W(J,2)
      B(J,1)  = B(J.2)
175   CONTINUE
      IF ((IP.LE.0).AND.((IP.NE.-2).OR.(ICYCLE.LT.ICYTL))) RETURN
C
C   PRINT OUT VELOCITY PROFILES
200   IF (NX.EQ.1)  GOTO 210
      IF (NX.LE.NS) THEN
         FAC1 = SQRT(X(NX)/RL/UE(NX))
         FAC2 = 1.0
      ELSE
         FAC1 = SQRT(X(NX)/RL)
         FAC2 = 1.0/UE(NX)
      ENDIF
      NPM1   = NP -1
      WRITE(6,4001) NX,X(NX)
      WRITE(6,4000)
      WRITE(6,4100) (J,ETA(J),F(J,2),U(J,2),V(J,2),W(J,2),B(J,2),
     +               ETA(J)*FAC1,U(J,2)*FAC2,J=1,NPM1,3)
      WRITE(6,4100) NP,ETA(NP),F(NP,2),U(NP,2),V(NP,2),W(NP,2),B(NP,2),
     +               ETA(NP)*FAC1,U(NP,2)*FAC2
C
210   IF (IP.NE.-2) RETURN
      IF ((NXVP(ICC,ISF).NE.NX).OR.(ICC.GT.NVP(ISF))) RETURN
      WRITE(12,4200) NP
      WRITE(12,4300) (ETA(J),J=1,NP)
      WRITE(12,4300) (U(J,2),J=1,NP)
      ICC = ICC+1
      RETURN
4001  FORMAT(/1H0,'NX =',I5,'  S/C =',F10.5)
4000  FORMAT(1H0,2H J,9X,3HETA,15X,1HF,13X,1HU,13X,1HV,13X,1HW,13X,1HB,
     +        13X,3HY/C,10X,4HU/UE)
4100  FORMAT(1H ,I3,E14.5,2X,5E14.5,2X,2E14.5)
4200  FORMAT(I5)
4300  FORMAT(8F10.6)
      END
C          DATA SET KCBCSMFIT  AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCSMFIT  AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCSMFIT  AT LEVEL 001 AS OF 08/15/83
      SUBROUTINE SMFIT(NS,ND,X,Q,D,KS)
C
C   THIS SUBROUTINE SMOOTHES DATA, Q, USING FIVE-POINT FORMULA.
C
C   NS  : BEGINNING POINT?   ND : END POINT
C   X   : INDEDENPENT COORDINATE? D  : WORKING STORAGE
C   Q   : VARIABLE TO BE SMOOTHED
C   KS  : NO OF SMOOTHING
      DIMENSION X(101),Q(101),D(101)
C   -------------------------------------------------------------
      SMT5(Q1,Q2,Q3,Q4,Q5    ) = 0.0625*(10.0*Q3+4.0*(Q2+Q4)-Q1-Q5)
      SMT3(Q1,Q2,Q3,X1,X2,X3) = 0.5*(Q2+(Q1*ABS(X3-X2)+Q3*ABS(X2-X1))
```

```
     +                                /ABS(X3-X1))
C
      IF(KS.LE.0) RETURN
      NSP1    = NS+1
      NSP2    = NS+2
      NDM1    = ND-1
      NDM2    = ND-2
C
      NDIF    = ND-NS+1
      IF ( NDIF .LT. 3 ) RETURN
      IF ( NDIF .LT. 5 ) GO TO 200
C
      DO 100 K=1,KS
      D(NS+1)= SMT3(Q(NS),Q(NS+1),Q(NS+2),X(NS),X(NS+1),X(NS+2))
      D(ND-1)= SMT3(Q(ND-2),Q(ND-1),Q(ND),X(ND-2),X(ND-1),X(ND))
      DO 20 I=NSP2,NDM2
      D(I)    = SMT5(Q(I-2),Q(I-1),Q(I),Q(I+1),Q(I+2))
20    CONTINUE
      DO 40 I=NSP1,NDM1
40    Q(I)      = D(I)
100   CONTINUE
      RETURN
C
200   DO 300 K = 1,KS
      DO 220 I = NSP1,NDM1
      D(I)    = SMT3(Q(I-1),Q(I),Q(I+1),X(I-1),X(I),X(I+1))
220   CONTINUE
      DO 250 I = NSP1,NDM1
      Q(I)    = D(I)
250   CONTINUE
300   CONTINUE
C
      RETURN
      END
C          DATA SET KCBCSOLV3  AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCSOLV3  AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCSOLV3  AT LEVEL 005 AS OF 02/21/84
      SUBROUTINE SOLV3
      COMMON /BLC0/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON /BLC1/ F(101,2),U(101,2),V(101,2),W(101,2),B(101,2)
      COMMON /BLC2/ DELF(101),DELU(101),DELV(101),DELW(101)
      COMMON /BLC6/ S1(101),S2(101),S3(101),S4(101),S5(101),S6(101),
     +                   S7(101),S8(101),R1(101),R2(101),R3(101),R4(101)
      COMMON /GRD / ETA(101),DETA(101),A(101)
      COMMON /BLCB/ A11(101),A12(101),A13(101),A14(101),
     +                   A21(101),A22(101),A23(101),A24(101)
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      A11(1)= 1.0
      A12(1)= 0.0
      A13(1)= 0.0
      A21(1)= 0.0
      A22(1)= 1.0
      A23(1)= 0.0
      G11=-1.0
      G12=-A(2)
      G13= 0.0
```

```
      G21= S4(2)
      G23=-S2(2)/A(2)
      G22= G23+S6(2)
C
C  FORWARD SWEEP
C
      DO 500 J=2,NP
      IF(J .EQ. 2) GO TO 100
      DEN   = (A13(J-1)*A21(J-1)-A23(J-1)*A11(J-1)-A(J)*
     +           (A12(J-1)*A21(J-1)-A22(J-1)*A11(J-1)))
      DEN1  = A22(J-1)*A(J)-A23(J-1)
      G11= (A23(J-1)+A(J)*(A(J)*A21(J-1)-A22(J-1)))/DEN
      G12= -(A(J)*A(J)+G11*(A12(J-1)*A(J)-A13(J-1)))/DEN1
      G13= (G11*A13(J-1)+G12*A23(J-1))/A(J)
      G21= (S2(J)*A21(J-1)-S4(J)*A23(J-1)+A(J)*(S4(J)*
     +           A22(J-1)-S6(J)*A21(J-1)))/DEN
      G22= (-S2(J)+S6(J)*A(J)-G21*(A(J)*A12(J-1)-A13(J-1)))/DEN1
      G23= G21*A12(J-1)+G22*A22(J-1)-S6(J)
100   A11(J)= 1.0
      A12(J)=-A(J)-G13
      A13(J)= A(J)*G13
      A21(J)= S3(J)
      A22(J)= S5(J)-G23
      A23(J)= S1(J)+A(J)*G23
      R1(J) = R1(J)-(G11*R1(J-1)+G12*R2(J-1)+G13*R3(J-1))
      R2(J) = R2(J)-(G21*R1(J-1)+G22*R2(J-1)+G23*R3(J-1))
500   CONTINUE
C
C  BACKWARD SWEEP
C
      DELU(NP) = R3(NP)
      E1       = R1(NP)-A12(NP)*DELU(NP)
      E2       = R2(NP)-A22(NP)*DELU(NP)
      DELV(NP) = (E2*A11(NP)-E1*A21(NP))/(A23(NP)*A11(NP)-A13(NP)*
     +              A21(NP))
      DELF(NP) = (E1-A13(NP)*DELV(NP))/A11(NP)
      J     = NP
600   J     = J-1
      E3    = R3(J)-DELU(J+1)+A(J+1)*DELV(J+1)
      DEN2  = A21(J)*A12(J)*A(J+1)-A21(J)*A13(J)-A(J+1)*A22(J)*A11(J)+
     +           A23(J)*A11(J)
      DELV(J)  = (A11(J)*(R2(J)+E3*A22(J))-A21(J)*R1(J)-E3*A21(J)*A12(J)
     +              )/DEN2
      DELU(J)  =-A(J+1)*DELV(J)-E3
      DELF(J)  = (R1(J)-A12(J)*DELU(J)-A13(J)*DELV(J))/A11(J)
      IF(J .GT. 1) GO TO 600
C
      DO 700 J=1,NP
      F(J,2)= F(J,2)+DELF(J)
      U(J,2)= U(J,2)+DELU(J)
      V(J,2)= V(J,2)+DELV(J)
700   CONTINUE
      U(1,2)= 0.0
      CALL EDGCHK(NP,ETA,F(1,2),U(1,2),V(1,2))
      RETURN
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

```
      END
C          DATA SET KCBCSOLV4  AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCSOLV4  AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCSOLV4  AT LEVEL 001 AS OF 02/21/84
      SUBROUTINE SOLV4(GAMMA1,GAMMA2)
      COMMON /BLC0/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON /BLC1/ F(101,2),U(101,2),V(101,2),W(101,2),B(101,2)
      COMMON /BLC2/ DELF(101),DELU(101),DELV(101),DELW(101)
      COMMON /BLC6/ S1(101),S2(101),S3(101),S4(101),S5(101),S6(101),
     +              S7(101),S8(101),R1(101),R2(101),R3(101),R4(101)
      COMMON /GRD / ETA(101),DETA(101),A(101)
      COMMON /BLCB/ A11(101),A12(101),A13(101),A14(101),
     +              A21(101),A22(101),A23(101),A24(101)
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
      A11(1)   = 1.0
      A12(1)   = 0.0
      A13(1)   = 0.0
      A14(1)   = 0.0
      A21(1)   = 0.0
      A22(1)   = 1.0
      A23(1)   = 0.0
      A24(1)   = 0.0
      DO 10 J = 2,NP
      AA1      = A13(J-1)-A(J)*A12(J-1)
      AA2      = A23(J-1)-A(J)*A22(J-1)
      AA3      = S2(J)-A(J)*S6(J)
      DET      = AA2*A11(J-1)-AA1*A21(J-1)
      AJS      = A(J)**2
      G11  = -(AA2+A21(J-1)*AJS)/DET
      G12  = (A11(J-1)*AJS+AA1)/DET
      G13  = A12(J-1)*G11+A22(J-1)*G12+A(J)
      G14  = A14(J-1)*G11+A24(J-1)*G12
      G21  = (S4(J)*AA2-A21(J-1)*AA3)/DET
      G22  = (A11(J-1)*AA3-S4(J)*AA1)/DET
      G23  = A12(J-1)*G21+A22(J-1)*G22-S6(J)
      G24  = A14(J-1)*G21+A24(J-1)*G22-S8(J)
      A11(J)   = 1.0
      A12(J)   = -A(J)-G13
      A13(J)   = A(J)*G13
      A14(J)   = -G14
      A21(J)   = S3(J)
      A22(J)   = S5(J)-G23
      A23(J)   = S1(J)+A(J)*G23
      A24(J)   = S7(J)-G24
      R1(J)    = R1(J) -G11*R1(J-1)-G12*R2(J-1)-R3(J-1)*G13
     +                 -G14*R4(J-1)
      R2(J)    = R2(J) -G21*R1(J-1)-G22*R2(J-1)-R3(J-1)*G23
     +                 -G24*R4(J-1)
   10 CONTINUE
C
C  BACKWARD SWEEP
      J        = NP
      G1       = GAMMA1/GAMMA2
      R3(J)    = R3(J)/GAMMA2
```

```
      R1(J)    = R1(J)-A12(J)*(R4(J)+R3(J))-A14(J)*R3(J)
      R2(J)    = R2(J)-A22(J)*(R4(J)+R3(J))-A24(J)*R3(J)
      C1       = A11(J)-G1*(A12(J)+A14(J))
      C2       = A21(J)-G1*(A22(J)+A24(J))
      DET      = C1*A23(J)-C2*A13(J)
      DELF(J)  = (R1(J)*A23(J)-R2(J)*A13(J))/DET
      DELV(J)  = (C1*R2(J)-C2*R1(J))/DET
      DELW(J)  = R3(J)-G1*DELF(J)
      DELU(J)  = R4(J)+DELW(J)
20    J        = J-1
      CC1      = DELU(J+1)-R3(J)-A(J+1)*DELV(J+1)
      CC2      = DELW(J+1)-R4(J)
      CC3      = A13(J)-A(J+1)*A12(J)
      CC4      = R1(J)-A12(J)*CC1-A14(J)*CC2
      CC5      = A23(J)-A(J+1)*A22(J)
      CC6      = R2(J)-A22(J)*CC1-A24(J)*CC2
      DENO     = A11(J)*CC5-A21(J)*CC3
      DELF(J)  = (CC4*CC5-CC3*CC6)/DENO
      DELV(J)  = (A11(J)*CC6-A21(J)*CC4)/DENO
      DELW(J)  = CC2
      DELU(J)  = CC1-A(J+1)*DELV(J)
      IF(J .GE. 2) GO TO 20
      DO 30 J = 1,NP
      F(J,2) = F(J,2)+DELF(J)
      U(J,2) = U(J,2)+DELU(J)
      V(J,2) = V(J,2)+DELV(J)
      W(J,2) = W(J,2)+DELW(J)
30    CONTINUE
      U(1,2) = 0.0
      CALL EDGCHK(NP,ETA,F(1,2),U(1,2),V(1,2))
      RETURN
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      END
C          DATA SET KCBCTRNS   AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCTRNS   AT LEVEL 001 AS OF 08/24/84
C          DATA SET KCBCTRNS   AT LEVEL 005 AS OF 03/13/84
      SUBROUTINE TRNS(ICODE)
C
C   CALCULATE TRANSITION LOCATION USING MICHEL CRITERION
C
      COMMON /BLC0/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON /BLC1/ F(101,2),U(101,2),V(101,2),W(101,2),B(101,2)
      COMMON/BLIN/ TITLE(20),XC(100),YC(100),ISG(100),DELS(100),
     +             XCTR,XTR,ISTRP,ICYCLE,ICYTL,XCTRS(2),TRFIND(2)
      COMMON/EDDY1/ RL,RX,SQRX,RXNTR,GMTR,GMTRS(100)
     +             ,ALFAS(100),FFS(100),RTS(100),IEDY,NXSPT
      COMMON /GRD / ETA(101),DETA(101),A(101)
      COMMON /GTY / X(101),UE(100),P1(100),P2(100),CEL,CELH
      COMMON/TRN/ PGAMTR,OMEGA,RTHETB,RTRANB
C   ------------------------------------------------------------
100   FORMAT(/3X,'BEGIN OF TRANSITION HAS BEEN DETECTED BY MICHEL''S ',
     +'CRITERION:  X/C =',F8.4,4X,'S/C =',F8.4,4X,'NTR =',I3/)
110   FORMAT(/3X,'BEGIN OF TRANSITION IS ASSUMED AT THE POINT OF ',
     +'LAMINAR SEPARATION:  X/C =',F8.4,4X,'S/C =',F8.4,4X,'NTR =',I3/)
C   ------------------------------------------------------------
      ICODE    = 0
```

```
      ISEP    = 1
C
      IF(V(1,2).LT. 0.0) THEN
C *** TRANSITION PROCESS HAS BEGUN DUE TO LAMINAR SEPARATION ***
         FAC = V(1,1)/(V(1,1)-V(1,2))
         GOTO 20
      END IF
C
C *** CHECK MICHEL'S TRANSITION CRITERION ***
      ISEP    = 0
      SUM     = 0.0
      F1      = U(1,2)/U(NP,2)*(1.0-U(1,2)/U(NP,2))
      DO 10 J=2,NP
      F2      = U(J,2)/U(NP,2)*(1.0-U(J,2)/U(NP,2))
      SUM     = SUM + (F1 + F2 ) *A(J)
      F1      = F2
   10 CONTINUE
      CONV    = SQRT(RL/X(NX))
      IF(NX.LE.NS) CONV = SQRT(RX)/X(NX)
      THETA   = SUM / CONV
      RTHETA = RL * UE(NX) * THETA
      RTRAN   = 1.174 * (1.0+22400.0/RX) * RX**0.46
      IF(RTHETA.LT.RTRAN) THEN
         RTHETB = RTHETA
         RTRANB = RTRAN
         RETURN
      END IF
C
C *** TRANSITION PROCESS HAS BEGUN BECAUSE OF MICHEL'S CRITERION ***
      FAC = (RTHETB-RTRANB)/(RTRAN-RTRANB-RTHETA+RTHETB)
C
C *** COMPUTE EXACT LOCATION OF TRANSITION BEGIN ***
   20 NTR   = NX-1
      NTR1 = NTR + 1
      XCTR = XC(NX-1) + FAC*(XC(NX)-XC(NX-1))
      XTR   = X(NX-1) + FAC*(X(NX)-X(NX-1))
      UETR = UE(NX-1) + FAC*(UE(NX)-UE(NX-1))
      IF ( ISEP .EQ. 0 ) WRITE (6,100) XCTR,XTR,NTR
      IF ( ISEP .EQ. 1 ) WRITE (6,110) XCTR,XTR,NTR
      ICODE   = 1
C
C *** CALCULATE INTERMITTENCY DISTRIBUTION ***
      RXNTR   = XTR * UETR * RL
      GGFT    = RL**2/PGAMTR/RXNTR**1.34*UETR**3
      DO 30 I=NTR1,NXT
      ALFAS(I) = 0.0168
      GMTRS(I)= 1.0
   30 CONTINUE
      ALFAS(NTR) = 0.0168
      UEINTG = 0.0
      U1 = 0.5/UETR
      X1 = XTR
      DO 40 I=NTR1,NXT
      U2 = 0.5/UE(I)
      X2 = X(I)
      UEINTG = UEINTG+(U1+U2)*(X2-X1)
```

```
      U1 = U2
      X1 = X2
      GG = GGFT*UEINTG*(X(I)-XTR)
      IF(GG .GT. 10.0) GOTO 50
      GMTRS(I) = 1.0-EXP(-GG)
   40 CONTINUE
C
C *** RESET FINITE DIFFERENCE CALCULATIONS ***
   50 DO 60 J=1,NPT
      F(J,2) = F(J,1)
      U(J,2) = U(J,1)
      V(J,2) = V(J,1)
      B(J,2) = B(J,1)
      W(J,2) = W(J,1)
60    CONTINUE
      RETURN
      END
C
C
      SUBROUTINE EDGCHK(NP, ETA, F, U, V)
C
      DIMENSION ETA(101), F(101), U(101), V(101)
C    -------------------------------------------------------
      JS     = NP - 3
      NPM1   = NP - 1
      DO 10 J=JS, NPM1
      JJ     = J
      IF(U(J).GE.U(NP) .OR.  V(J).LT.0.0) GOTO 20
10    CONTINUE
      RETURN
20    JS = JJ - 1
      IF(JS.GT.(NP-2)) JS = NP-2
      CALL AMEAN(JS, NP, ETA, U, 1)
      CALL AMEAN(JS, NP, ETA, F, 1)
      DETAP  = ETA(JS) -ETA(JS-1)
      VJP    = (U(JS)-U(JS-1))/DETAP
      DO 30 J=JS,NPM1
      DETAM  = ETA(J+1)-ETA(J)
      VJM    = (U(J+1)-U(J))/DETAM
      V(J)   = (VJM*DETAP + VJP*DETAM)/(DETAP+DETAM)
      VJP    = VJM
      DETAP  = DETAM
30    CONTINUE
      V(NP)  = -V(NP-1) + 2.0 * VJP
      RETURN
C   ******************************************************************
C   NOTES:         (FOR CHANGING FROM THE ORIGINAL PROGRAM)          *
C                                                                    *
C   1.  'EDDY' HAS BEEN MODIFIED BY ADDING 'FINT'.                   *
C   2.  SUBROUTINE 'EDGCHK' HAS BEEN ADDED.                          *
C   3. GROWTH LIMIT HAS BEEN ADDED FOR 2 IN 'MAIN2'.                 *
C                                                                    *
C                                                                    *
C   ******************************************************************
      END
```

```
      SUBROUTINE SMPSON
C
      COMMON /BLC0/ NX,NXT,NP,NPT,NTR,IT,INVRS,NS,IP
      COMMON /BLC1/ F(101,2),U(101,2),V(101,2),W(101,2),B(101,2)
      COMMON /BLC7/ C(100,100),D(100),DB(100),DBP(100),UE0(100),GI
      COMMON/EDDY1/ RL,RX,SQRX,RXNTR,GMTR,GMTRS(100)
     +                  ,ALFAS(100),FFS(100),RTS(100),IEDY,NXSPT
      COMMON /GTY / X(101),UE(100),P1(100),P2(100),CEL,CELH
      COMMON /GRD / ETA(101),DETA(101),A(101)
      DIMENSION CRD(12),RTD(12)
      DATA RTD/0.00,0.05,0.12,0.20,0.30,0.40,0.50,0.60,0.70,
     +              0.80,0.90,1.00/
      DATA CRD/5.00,4.75,4.35,3.80,3.25,2.85,2.58,2.37,2.25,
     +              2.15,2.06,2.00/
C
C    -------------------------------------------------------
C
C    STEP 1 CALCULATE (DU/DX)/(DU/DY)
C      IF(NX.LT.NXSPT) GOTO 10
C
C    IN THE SEPARATED REGION, ALFA SET TO BE CONSTANT
C      ALFAS(NX)= ALFASP
C      RETURN
C(-----
C
C  10 CONTINUE
C      IF(V(1,2).GT. 0.0) GOTO 20
C
C    SEPARATION OCCURS. ALFA SET TO BE THE PREVIOUS ITERATED VALUE
C      ALFASP = ALFAS(NX)
C      NXSPT  = NX
C      RETURN
C(-----
C    MODIFY OUTER EDDY BASED ON SIMPSON SUGGESTION
      TM     = 0.0
      JM     = 1
      DO 30 J=2,NP
      TS     = (B(J,2)-1.0)* V(J,2)
      IF(TS.LT.TM) GOTO 30
      TM     = TS
      JM     = J
30    CONTINUE
      VNXM = 0.5*(V(JM,2)+V(JM,1))
      IF (NX .LE. NS) GOTO 35
      DUDX = (U(JM,2)-U(JM,1)) / (X(NX)-X(NX-1))
      GO TO 38
35    DUDX   = CEL*(U(JM,2)-U(JM,1))+P2(NX)*U(JM,2)+0.5*ETA(JM)*
     +              VNXM*(P2(NX)-1.0)
38    RU     = RL
      IF(NX.LE.NS)RU = RL * UE0(NX) * X(NX)
      RL2    = SQRT(RU)
      RR     = DUDX/VNXM/RL2
C
C    STEP 2 : CALCULATE (UU - VV)/UV
      VNXM = 0.5*(V(1,2)+V(1,1))
```

```
      RT     = VNXM/TM
C    PRINT'(3X,2I5,3F10.3)',NX,JM,VNXM,TM,RT
      IF (RT .LT.  0.0) RT = 0.0
      IF(RT.GT.1.0) GOTO 60
      CR     = 6.0 /(1.0 +  2.0 * RT*(2.0 -RT))
C     CR     = 2.0
C     DO 40 I=2,12
C     IF(RT.LT.RTD(I)) GOTO 50
C  40 CONTINUE
C     GOTO 70
C  50 CR     = CRD(I-1)+(CRD(I)-CRD(I-1))*(RT-RTD(I-1))/(RTD(I)-RTD(I-1))
      GOTO 70
60    CR     = (1.0 + RT ) /RT
C
C    STEP 3 : CALCULATE FF
70    FR     = CR * RR
      IF(FR .GT.0.35) FR = 0.35
      IF (FR .LT.  -0.8) FR = -0.8
      FFS(NX)= (FFS(NX) +  (1.0 -FR))/ 2.0
      RTS(NX)= RT
      ALFAS(NX)= 0.0168/FFS(NX)**2.5
      RETURN
C(-----
      END
C
      SUBROUTINE XSPACE(NI,NRITE,XII,XLLT,RAD,NL1,NR1)
      DIMENSION XII(200),T(200)
      DATA PI/3.14159265359879/
      RAD = PI
      NLEFT=NI-1-NRITE
      NR4=NRITE/2
      IF((NRITE/2*2) .NE. NRITE) NR4=(NRITE+1)/2
      NL1=NR4+1
      NL2=NR4+NLEFT
      NR1=NL2+1
      NR2=NI
      PI2=0.5*PI
      RAD2=(PI-RAD)/2.0+PI2
      RAD3=RAD2+RAD
      SRT =RAD2/FLOAT(NR4)
      SRT2=SRT
      IF((NRITE/2*2) .NE. NRITE) SRT2=RAD2/FLOAT(NR4-1)
      SLT = RAD/FLOAT(NLEFT)
      DO 10 I=1,NR4
10    XII(I)=0.5*(1.0+COS(FLOAT(I-1)*SRT))
      DO 20 I=NL1,NL2
20    XII(I)=0.5+XLLT*COS(FLOAT(I-NL1)*SLT+RAD2)
      DO 30 I=NR1,NR2
30    XII(I)=0.5*(1.0+COS(FLOAT(I-NR1)*SRT2+RAD3))
      NA=(NI+1)/2
      IF((NI/2*2) .EQ. NI) NA=NI/2+1
      FN1=FLOAT(NA-1)
      FN2=FN1
      IF((NI/2*2) .EQ. NI) FN2=FLOAT(NA-2)
      DO 40 I=1,NA
40    T(I)=FLOAT(NA-I)/FN1
```

```
      CALL AMEAN(1,NA,T,XII,1)                                          INT28390
      XDIF = XII(1) - XII(2)                                            INT28400
      IF(XDIF .LT. 0.004) THEN                                          INT28410
      DO 45 I=2,5                                                       INT28420
      XII(I) = XII(I-1)-XDIF*3.0                                        INT28430
45    CONTINUE                                                          INT28440
      CALL AMEAN(2,NA,T,XII,10)                                         INT28450
      END IF                                                            INT28460
      DO 50 I=NA,NI                                                     INT28470
50    XII(I) = XII(NI-I+1)                                              INT28480
      RETURN                                                            INT28490
      END                                                               INT28500
      SUBROUTINE TRGRID (N1 , XO , YO,NI,NRITE,XLLT,N1O,RAD,ID,NXSS)    INT28510
C  THIS SUB. IS TO REGRID SPACING NEAR TRAILING-EDGE                    INT28520
C                                                                       INT28530
      DIMENSION XO(200),YO(200),XI(200),YI(200),D1(200),D2(200),D3(200),INT28540
     +          XOO(200),YOO(200),XII(200),YII(200),WX(200),WY(200),    INT28550
     +          WXI(200),WYI(200),T(200)                                INT28560
C                                                                       INT28570
      N2O    =  N1O                                                     INT28580
      IF((N1/2*2) .EQ. N1) N2O = N1O+1                                  INT28590
      IF((NI-(NI/2)*2) .NE. 0) N1I= (NI-1)/2+1                          INT28600
      IF((NI-(NI/2)*2) .EQ. 0) N1I=NI/2                                 INT28610
      N2I    =  N1I                                                     INT28620
      IF((NI/2*2) .EQ. NI) N2I = N1I+1                                  INT28630
C                                                                       INT28640
      CALL XSPACE(NI,NRITE,XI,XLLT,RAD,NL1,NR1)                         INT28650
C      PRINT *,'NRITE=',NRITE,' XLLT=',XLLT                             INT28660
C     WRITE (6, 290)                                                    INT28670
C     WRITE (6, 300) (XO(I) ,I=1,N1)                                    INT28680
C     WRITE (6, 298)                                                    INT28690
C     WRITE (6, 300) (YO(I) ,I=1,N1)                                    INT28700
      IF(ID .EQ. 2) THEN                                                INT28710
      DO 60 I=NL1,NXSS                                                  INT28720
      YI(I)=YO(I)                                                       INT28730
60    XI(I)=XO(I)                                                       INT28740
      NXST=NXSS+8                                                       INT28750
      XM1=(XI(NXST)-XI(NXSS))/8.0                                       INT28760
      XM2=(XI(NR1)-XI(NXSS))/(NR1-NXSS)                                 INT28770
      DO 62 I=NXSS,NR1-1                                                INT28780
62    XI(I)=(XO(I)+XI(I))/2.0                                           INT28790
      DO 65 I=NXSS,NXST                                                 INT28800
65    T(I)=FLOAT(I-NXSS)*XM1+XI(NXSS)                                   INT28810
      CALL AMEAN(NXSS,NXST,T,XI,8)                                      INT28820
      DO 68 I=NXSS,NR1                                                  INT28830
68    T(I)=FLOAT(I-NXSS)*XM2+XI(NXSS)                                   INT28840
      CALL AMEAN(NXSS,NR1,T,XI,12)                                      INT28850
      N1O=NL1                                                           INT28860
      N1I=N1O                                                           INT28870
      N2O=N1+1-NXSS                                                     INT28880
      N2I=NI-NXSS+1                                                     INT28890
      ELSE                                                              INT28900
      NXSS=N2I                                                          INT28910
      END IF                                                            INT28920
C  FOR LOWER SURFACE                                                    INT28930
      DO 5 I = 1 , N1O                                                  INT28940
```

```
      II = N10 - I + 1
      WX(II) = XO(I)
      WY(II) = YO(I)
5     CONTINUE
      DO 7 I = 1 , N1I
      II = N1I - I + 1
      WXI(II) = XI(I)
7     CONTINUE
      CALL DIFF3(N10,WX,WY,D1,D2,D3,0)
      CALL INTRP3(N10,WX,WY,D1,D2,D3,N1I,WXI,WYI)
      DO 9 I = 1 , N1I
      II = N1I - I + 1
      YI(II) = WYI(I)
9     CONTINUE
C
C  FOR UPPER SURFACE
      DO 10 I = 1 , N20
      II = N1 - N20 + I
      XOO(I) = XO(II)
      YOO(I) = YO(II)
10    CONTINUE
      DO 20 I = 1 , N2I
      II = NI - N2I + I
      WXI(I) = XI(II)
20    CONTINUE
      CALL DIFF3(N20,XOO,YOO,D1,D2,D3,0)
      CALL INTRP3(N20,XOO,YOO,D1,D2,D3,N2I,WXI,WYI)
      DO 25 I = 1 , N2I
      II = N2I - I + 1
      XII(II) = WXI(I)
      YII(II) = WYI(I)
25    CONTINUE
C
C  COMBINE TWO SURFACES INTO ONE CIRCLE
C
C     NN = N1 - N10 - N20
C     DO 30 I = 1 , NN
C     II = N1I + I
C     III= N10 + I
C     XI(II) = XO(III)
C     YI(II) = YO(III)
C30   CONTINUE
      DO 40 I = 1 , N2I
      I1 = NXSS + I -1
      I2 = N2I - I + 1
      XI(I1) = XII(I2)
      YI(I1) = YII(I2)
40    CONTINUE
      XI(1) = XO(1)
      XI(I1)= XO(N1)
      YI(1) = YO(1)
      YI(I1)= YO(N1)
C
      N1 = I1
      DO 50 I = 1 , N1
      XO(I) = XI(I)
```

```
      YO(I) = YI(I)
50    CONTINUE
C
C     WRITE (6 , 295)
C     WRITE (6 , 300) (XO(I) , I=1,N1)
C     WRITE (6 , 298)
C     WRITE (6 , 300) (YO(I) , I=1,N1)
C
      RETURN
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
100   FORMAT(7I5)
200   FORMAT(6F10.0)
290   FORMAT(/,'  ORIGINAL COORDINATES',/,'  X/C')
295   FORMAT(/,'  INTERPLATED COORDINATES',/,'  X/C')
298   FORMAT('     Y/C')
300   FORMAT(6F10.6)
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      END
C
      SUBROUTINE STAGR(N,STAG,XO,YO,XSTGR,YSTGR)
C
      DIMENSION XO(100),YO(100),XSTGR(100),YSTGR(100),DS(100)
C
      XOTE = 0.5 * (XO(1)+XO(N))
      YOTE = 0.5 * (YO(1)+YO(N))
      DS(1) = SQRT((XO(1)-XOTE)**2 + (YO(1)-YOTE)**2)
      DSM = DS(1)
      DO 10 I = 2 , N
      DS(I) = SQRT((XO(I)-XOTE)**2 + (YO(I)-YOTE)**2)
      IF (DS(I) .LT. DSM) GOTO 10
      IM = I
      DSM = DS(I)
10    CONTINUE
C
      YYY = YOTE-YO(IM)
      XXX = XOTE-XO(IM)
      IF (YYY .EQ. 0.0 .AND. XXX .EQ. 0.0) THEN
      ANG = 0.0
      ELSE
      ANG = ATAN2(YYY,XXX)
      END IF
      ANG = ANG + STAG
C
      COSAN = COS(ANG)
      SINAN = SIN(ANG)
      DO 20 I = 1 , N
C     YY = YO(I)-YO(IM)
C     XX = XO(I)-XO(IM)
C     IF (YY .EQ. 0.0 .AND. XX .EQ. 0.0) THEN
C     ANGCO = 0.0
C     ELSE
C     ANGCO = ATAN2(YY,XX)
C     END IF
      XSTGR(I)= XO(I)*COSAN + YO(I)*SINAN
      YSTGR(I)= YO(I)*COSAN - XO(I)*SINAN
20    CONTINUE
```

```
      RETURN                                                      INT30070
      END                                                         INT30080
```

APPENDIX B. C4 CASCADE

A. EXPERIMENTAL RESULTS

The experimental results of the C4 cascade were obtained directly from professor G.J. Walker, University of Tasmania, Tasmania, Australia, who performed these experiments.

The results of the boundary layer measurements of the C4 cascade are given below at four inlet angles: 34.1°, 36.3°, 45.6°, and 47.7°. The Reynold numbers, based on the chord and the upstream velocity, are 200000, 191000, 173000 and 171000 respectively. The results given in the following tables include the displacement thickness (δ^*), the shape factor (H) and the local free stream velocity (UE).

Table 1. EXPERIMENTAL RESULTS AT INLET ANGLE OF 34.1°

x/c	δ^* [10^{-3} FT]	H	UE [FT/SEC]
0.4	4.9	2.48	168.37
0.5	6.28	2.61	167.35
0.6	8.79	3.24	158.31
0.7	10.83	3.63	149.27
0.8	16.63	3.79	147.13
0.9	16.19	1.89	143.79

Table 2. EXPERIMENTAL RESULTS AT INLET ANGLE OF 36.3°

x/c	δ^* [10^{-4} FT]	H	UE [FT/SEC]
0.4	5.43	2.55	161.63
0.5	7.09	2.70	157.59
0.6	10.3	3.34	148.78
0.7	12.63	3.78	139.87
0.8	14.84	2.78	135.01
0.9	16.43	1.76	133.23

Table 3. EXPERIMENTAL RESULTS AT INLET ANGLE OF 45.6°

x c	δ^* [10^{-3} FT]	H	UE [FT SEC]
0.4	8.08	2.58	137.88
0.5	9.83	2.41	133.70
0.6	12.35	2.33	122.18
0.7	12.98	1.97	114.93
0.8	19.44	1.90	111.77
0.9	27.69	1.92	109.26

Table 4. EXPERIMENTAL RESULTS AT INLET ANGLE OF 47.7°

x c	δ^* [10^{-4} FT]	H	UE [FT SEC]
0.4	8.87	2.24	130.64
0.5	10.27	2.19	124.76
0.6	14.31	2.08	116.86
0.7	16.45	1.87	106.72
0.8	24.16	1.82	103.75
0.9	36.00	2.01	102.18

The results of the measurements of the velocity profiles in the boundary layer at two inlet angles, 34.1° and 36.3° at 50% chord are given below.

Table 5. VELOCITY PROFILES AT 50% CHORD.

y	β = 36.3°	β = 34.1°
0.0	0.0	0.0
2.3	0.172	0.208
3.7	0.270	0.327
6.2	0.469	0.534
8.6	0.666	0.728
11.0	0.794	0.867
13.4	0.891	0.933
18.3	0.982	0.985
23.2	1.000	1.000

B. C4 CASCADE COORDINATES

```
      DIMENSION X(0:100),XU(0:100),XL(0:100),YU(0:100),YL(0:100)
      DATA A1,A2,A3,A4/0.15492,0.06563,0.2528,0.2811/
      DATA B1,B2,B3,B4/0.03866,0.07871,0.1467,0.03448/
      PI = ACOS(-1.0)
C     READ (5,800) NMAX
  800 FORMAT (I5)
      NMAX=33
C     READ (5,810) (X(I),I=0,NMAX)
  810 FORMAT (6F10.6)
      DO 50 I=0,NMAX
      X(I) = (1.0-COS(PI*I/NMAX))/2.
   50 CONTINUE
      DO 100 I=0,NMAX
      SRT = SQRT((0.5/SIN(PI/12))**2-(0.5-X(I))**2)
      YC = -0.5/TAN(PI/12) + SRT
      DY = ATAN((0.5-X(I))/SRT)
      IF (X(I).LT.0.3) THEN
         YT = A1*SQRT(X(I)) - A2*X(I) - A3*X(I)**2 + A4*X(I)**3
      ELSE
         YT = B1 + B2*X(I) - B3*X(I)**2 + B4*X(I)**3
      END IF
```

```
      YU(I) = YC + COS(DY)*YT
      YL(I) = YC - COS(DY)*YT
      XU(I) = X(I) - SIN(DY)*YT
      XL(I) = X(I) + SIN(DY)*YT
  100 CONTINUE
C     WRITE (6,900) (I,X(I),XU(I),YU(I),XL(I),YL(I),I=0,NMAX)
C 900 FORMAT (I5,4X,F10.6,4X,2F10.6,4X,2F10.6)
      WRITE (1,910) (XL(I),I=NMAX,0,-1),(XU(I),I=1,NMAX)
      WRITE (1,910) (YL(I),I=NMAX,0,-1),(YU(I),I=1,NMAX)
  910 FORMAT (6F10.6)
      STOP
      END
```

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